

ANALÝZA MECHANISMŮ S VÍCE STUPNI VOLNOSTI

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ANALYSIS OF MECHANISMS WITH MORE DEGREES OF FREEDOM

Abstract: *At CTU in Prague was developed the tool „Sungear“ dedicated for analysis of planetary gearboxes consisting of single or nested planetary gearsets; and for analysis of the powersplit CVT or IVT mechanisms with 1 degree of freedom (DOF). In this paper is briefly described the code and its further evolution, which helps to determine the kinematics and torque distribution in mechanisms with more degrees of freedom.*

Key words: *Sungear, differential, matrix method, power-split transmissions*

INTRODUCTION

The calculation of kinematic and energetic parameters of planetary systems and mechanisms with powersplit is well known and does not present any problems. The matrix method for calculation of planetary gear sets was introduced by Prof. Šalamoun in the 70's of last century [1]. The matrix method is based on the transformed Willis formulae [5]. The rearrangement of the equation is made in such a manner, that there only the basic ratio of the planetary gear set appears, i.e. ratio with fixed axes. The method was deduced for simple planetary sets only. In 2007 this method was further elaborated for calculation of reduced or Ravigneaux gear sets [2]. The Ravigneaux gear set was split into two single planetary sets (the link is always a spider), and further the calculation proceeds as for any nested planetary set consisting of single planetary sets.

The matrix method was later completed with procedure how to analyse the powersplit mechanisms consisting of different types of transmissions (i.e. hydrostatic or hydrodynamic transmission, CVT'S, chain drives, gearwheel ratios etc.), where the different transmission systems are connected together with help of differential to a functional powersplit 1 DOF system, where the advantages of the used mechanisms could be combined.

The power of matrix method lies in its easy implementation in computer calculations. Based on the matrix method we developed a new program „SunGear“ which is dedicated for calculation of powersplit systems and complete analysis of mechanical stepped planetary gearboxes. Its description is a part of this article also.

The added value of this article is implementation of calculation of more degree of freedom differentials.

THEORETICAL BACKGROUND

All known methods for calculating kinematics of planetary gear sets are based on the Willis formulae. The following equation represents the Willis formula describing the kinematics of a mechanism with 2 DOF's. The indexes x , y and z belong to whichever member of planetary set (sun, ring or spider).

$$\omega_x - i_{x,y}^z \cdot \omega_y - i_{x,z}^y \cdot \omega_z = 0 \quad (1)$$

The calculation of the ratio or estimate the losses in gear mesh is possible only for mechanisms where all members have fixed axes. That's not the case for planetary gear sets, where the planets move with epicyclic movement around the central axes of the planetary set. The only state, where all gearwheels have fixed axes is when the spider is fixed. The rearrangement of equation (1) can be done with help of Willis theorem (2).

$$i_{x,y}^z = \frac{\omega_x - \omega_z}{\omega_y - \omega_z} \quad \omega_y \text{ is fixed} \Rightarrow \omega_y = 0 \Rightarrow i_{x,y}^z = -\frac{\omega_x - \omega_z}{\omega_z} = 1 - \frac{\omega_x}{\omega_z} \quad (2)$$

$$i_{x,y}^z = 1 - i_{x,z}^y$$

The transformed Willis formula for mechanisms with 2 DOF has the following form – see equation (3). Instead of the general index „z“ the index „r“ specific for the spider is used.

$$\omega_x - i_{x,y}^r \cdot \omega_y - (1 - i_{x,y}^r) \cdot \omega_r = 0 \Rightarrow \omega_x - i_{x,y}^r \cdot \omega_y + (i_{x,y}^r - 1) \cdot \omega_r = 0 \quad (3)$$

Writing the transformed Willis formulae for each set of nested planetary gear set, we obtain the system of linear algebraic equations, i.e. we obtain the matrix of kinematics.

The matrix of kinematics can be used also for estimation of the total efficiency of the mechanism (the gearing losses are taken into account only). Introducing the efficiency η^r defined in the system of coordinates connected to a spider (mechanism with fixed axis). To determine the efficiency exponent is used the method derived by Krejnes in [4] and presented in equation (4).

$$\exp = \operatorname{sgn}\left(\frac{i^r}{i}\right) \cdot \operatorname{sgn}\left(\frac{\partial i}{\partial i^r}\right) = \operatorname{sgn}\left(\frac{i^r}{i} \cdot \frac{\partial i}{\partial i^r}\right) \quad (4)$$

For momentary parameters we can, by analogy with kinematical parameters, set the system of algebraic homogenous equations. Every planetary gear set is described by two

momentary equations – one resulting from the equation of power equilibrium (5) and a second resulting from the equation of momentary equilibrium (6) on the planetary set. The equations are derived for single planetary set consisting of two gearwheels x and y and spider r .

$$\sum P = 0 \Rightarrow P_x + P_y + P_r = 0 \quad (5)$$

$$M_x \cdot \omega_x + M_y \cdot \omega_y + M_r \cdot \omega_r = 0$$

$$\omega_r = 0 \Rightarrow M_x \cdot \omega_x + M_y \cdot \omega_y = 0$$

$$M_x \cdot i_{x,y}^r + M_y = 0$$

$$\sum M = 0 \Rightarrow M_x + M_y + M_r = 0 \quad (6)$$

$$M_x - M_x \cdot i_{x,y}^r + M_r = 0$$

$$M_x \cdot (1 - i_{x,y}^r) + M_r = 0$$

Further we need to introduce the equations of momentary equilibrium of external and internal linkage shafts.

For calculation of powersplit mechanisms consisting of non-geared transmissions the analogy with the above described equations is used. These transmissions are generally represented by characteristic and not by constant value.

SUNGEAR

Accordingly to the explanations stated in previous section, the program SunGear was developed in the frame of Master thesis work [7] in the Matlab[®] environment, with use of additional Symbolic Math Toolbox[™]. The program allows the complete analysis of almost all types of powersplit mechanisms. The starting window offers the choice of calculation of:

- Planetary gearbox
- Powersplit mechanism
- Differentials.

The planetary gearbox can be compound from four single planetary gear sets. The challenging fact is that single planetary gear set can be designed in various forms. A new system how to find an algorithmisation of this problem was found. Lévai [6] categorized 12 mostly used simple planetary gear sets – see Figure 1. Plus were included the schemes of united planetary gear sets. Every planetary gear set was drawn on one layer. By determining the overlap of all layers the scheme depicted below on Figure 1 was found.

Basic gear ratio, other gear ratios of planet gears, basic efficiencies and assembly condition for uniformly distributed planets are constantly recalculated after any change in settings of single planetary set.

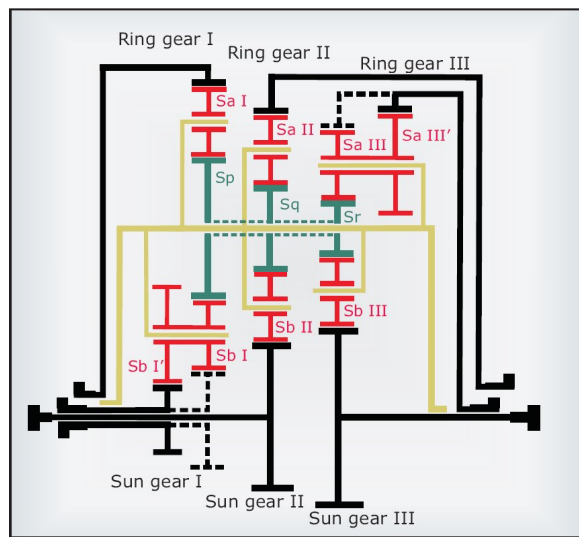
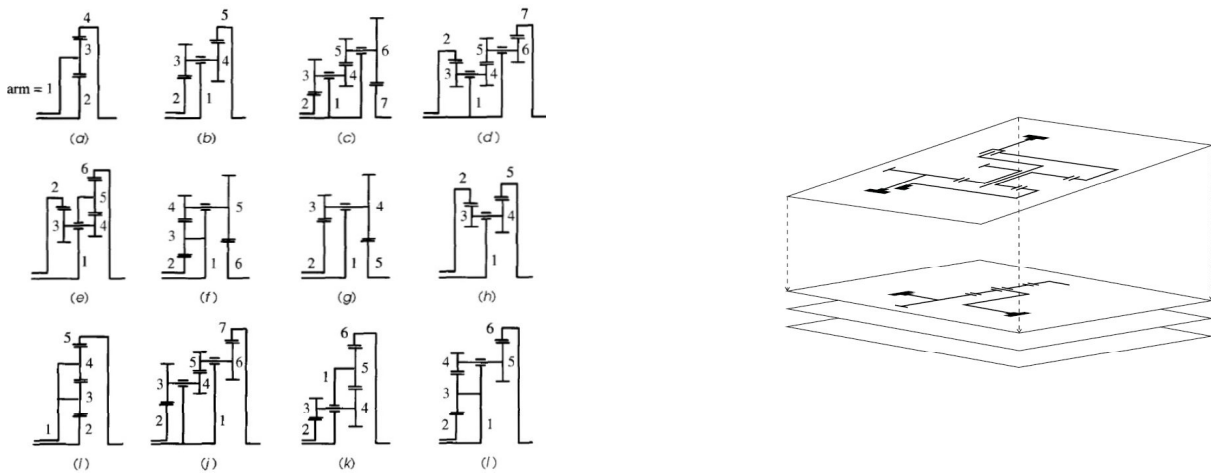


Fig 1: The procedure, how the general scheme was found, which should cover the schemes of all existing single planetary sets. Upper figure represents the Lévai's [6] definition of 12 mostly used PGSS; next figure represents their overlap (the united PGSS completed the Levais definition); last figure results in general scheme.

After definition of the number of nested planetary gear sets and its composition, the working scheme for different speeds can be defined. For every planetary set corresponds a panel, which comprises a set of control objects – see Figure 2.

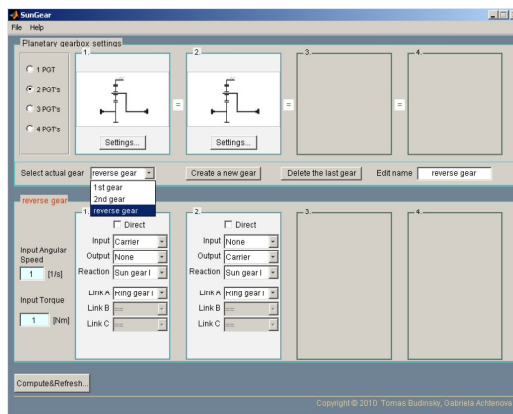


Fig 2: Window for definition of actual connection of PGS's of planetary mechanism for the chosen speed.

The results are obtained in the output window – see Figure 3, where on the left side the graph of relative angular speeds of all elements on every engaged speed and on the right side an overview of all computed values in form of a table is placed.

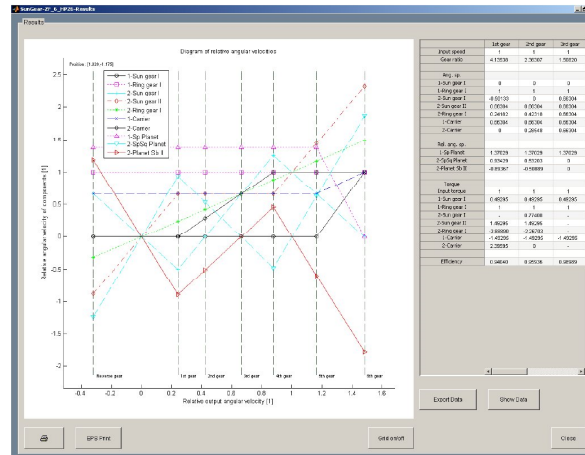


Fig 3: The output window comprises graph of relative speeds on all elements and table with computed parameters for all speeds.

For calculation of powersplit transmissions a new panel appears – as shown in Figure 4. Two possibilities of system are offered: mechanisms with differential linked to the input shaft of the mechanism, and differential linked to the output shaft of the mechanism.

For both input and output split mechanism, the CVT unit can be either purely mechanical, hydrodynamic or hydrostatic. The mechanical CVT unit has to be defined by its operational limits and efficiency, which is considered constant.

In contrast, the efficiency of a hydrodynamic or hydrostatic CVT is not constant in its operational range. Therefore, it has to be defined depending on the operational range. The data import is provided. The input data are written in *.mat files (produced by MATLAB environment).

An example of calculated powersplit transmission with combination of planetary gear set and torque converter is depicted in Figure 4. The planetary differential is placed on the input of the whole mechanism. As input parameter serve a table with kinematic coefficient and efficiency of the torque converter, basic ratio of the differential and ratio of the linkage gear.

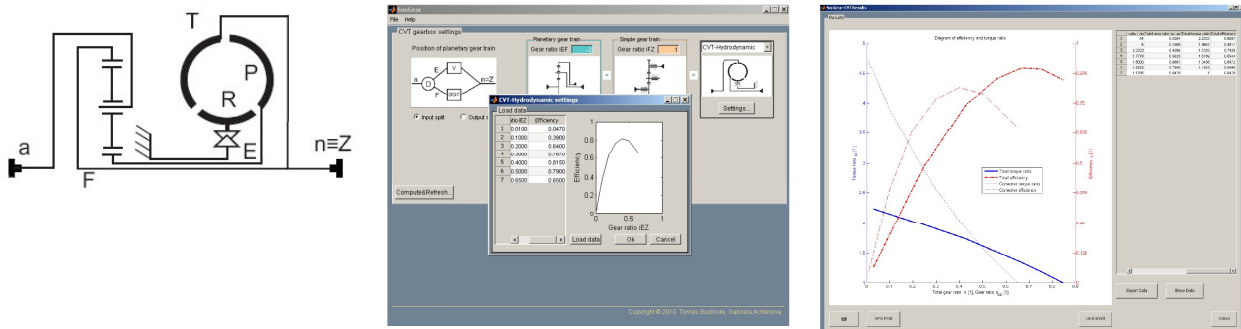


Fig 4: Example of calculation of powersplit system with torque converter. The scheme of the mechanism is depicted on the left side. In the middle there is input window. Right is an overview of calculated results.

ANALYSIS OF DIFFERENTIAL WITH MORE DEGREES OF FREEDOM

So far we described the case of simple planetary set which can have maximally 2 DOF's – its kinematics is described with help of equation (1). The differentials can be more complex mechanisms having more external shafts and more DOF's. In the following text we will estimate the number of independent equations and input parameters for whichever mechanism with more DOF's and more external shafts.

In a mechanism with s external shafts there exist s kinematical parameters and s torques. So totally there are $2 \cdot s$ parameters. But not all of them are independent, i.e. not all can be chosen. The number of independent kinematical input parameters equals DOF. The number of preassigned torques equals the number of external shafts minus DOF. The number of independent parameters equals the number of external shafts s .

We can write e_ω kinematical equations and we can write e_t equations of torque equilibrium.

$$e_\omega = s - DOF \quad (7)$$

$$e_t = s - e_\omega \quad (8)$$

We will have a differential (2 DOF) with 4 external shafts marked x, y, w, z . The number of kinematical parameters we should chose (or should be known) = DOF. Let's suppose we will know rotational speed of shaft x and w . To calculate the remaining 2 speeds we will use Willis formula (1).

$$\begin{aligned} \omega_y &= i_{y,x}^w \cdot \omega_x + i_{y,w}^x \cdot \omega_w \\ \omega_z &= i_{z,x}^w \cdot \omega_x + i_{z,w}^x \cdot \omega_w \end{aligned} \quad (9)$$

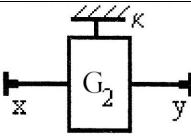
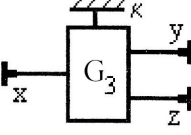
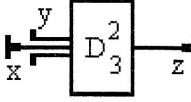
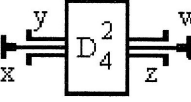
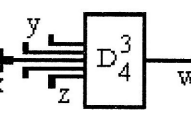
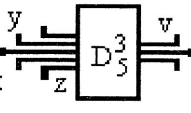
We need two equations of torque equilibrium for two known parameters. We can write the equation of power equilibrium. When introducing the two kinematic equations (9) in the

equation of power equilibrium we will obtain the equations of momentary equilibrium (the efficiency is not taken into account).

$$\begin{aligned} M_x + M_y \cdot i_{y,x}^w + M_z \cdot i_{z,x}^w &= 0 \\ M_w + M_y \cdot i_{y,w}^x + M_z \cdot i_{z,w}^x &= 0 \end{aligned} \quad (10)$$

The derivation of equations of kinematics and of torque equilibrium is made and summarised in the following table.

Tab 1: Overview of kinematical and torque equilibrium equations for chosen types of mechanisms, [8].

Scheme	Kinematic equation	Torque equilibrium
	$\omega_y = i_{y,x}^K \cdot \omega_x$	$M_x + i_{y,x}^K \cdot M_y = 0$ $M_x + i_{K,x}^y \cdot M_K = 0$
	$\omega_y = i_{y,x}^K \cdot \omega_x$ $\omega_z = i_{z,x}^K \cdot \omega_x$	$M_x + i_{y,x}^K \cdot M_y + i_{z,x}^K \cdot M_z = 0$ $M_K + i_{z,K}^y \cdot M_z + i_{y,x}^K \cdot M_y = 0$
	$\omega_x = i_{x,y}^z \cdot \omega_y + i_{x,z}^y \cdot \omega_z$	$M_x + i_{z,x}^y \cdot M_z = 0$ $M_y + i_{z,y}^x \cdot M_z = 0$
	$\omega_y = i_{y,x}^w \cdot \omega_x + i_{y,w}^x \cdot \omega_w$ $\omega_z = i_{z,x}^w \cdot \omega_x + i_{z,w}^x \cdot \omega_w$	$M_x + M_y \cdot i_{y,x}^w + M_z \cdot i_{z,x}^w = 0$ $M_w + M_y \cdot i_{y,w}^x + M_z \cdot i_{z,w}^x = 0$
	$\omega_w = i_{w,x}^{y,z} \cdot \omega_x + i_{w,y}^{x,z} \cdot \omega_y + i_{w,z}^{x,y} \cdot \omega_z$	$M_x + i_{w,x}^{y,z} \cdot M_w = 0$ $M_y + i_{w,y}^{x,z} \cdot M_w = 0$ $M_z + i_{w,z}^{x,y} \cdot M_w = 0$
	$\omega_v = i_{v,x}^{y,z} \cdot \omega_x + i_{v,y}^{x,z} \cdot \omega_y + i_{v,z}^{x,y} \cdot \omega_z$ $\omega_w = i_{w,x}^{y,z} \cdot \omega_x + i_{w,y}^{x,z} \cdot \omega_y + i_{w,z}^{x,y} \cdot \omega_z$	$M_x + i_{w,x}^{y,z} \cdot M_w + i_{v,x}^{y,z} \cdot M_v = 0$ $M_y + i_{w,y}^{x,z} \cdot M_w + i_{v,y}^{x,z} \cdot M_v = 0$ $M_z + i_{w,z}^{x,y} \cdot M_w + i_{v,z}^{x,y} \cdot M_v = 0$

The equations summarized in the table are programmed in the Sungear as next possibility. The speed of rotation can be given as constant or as field of linearly distributed values between given limits. The torque can be introduced as constant value only. Based on the type of given parameters the results are printed as constant values or as graph.

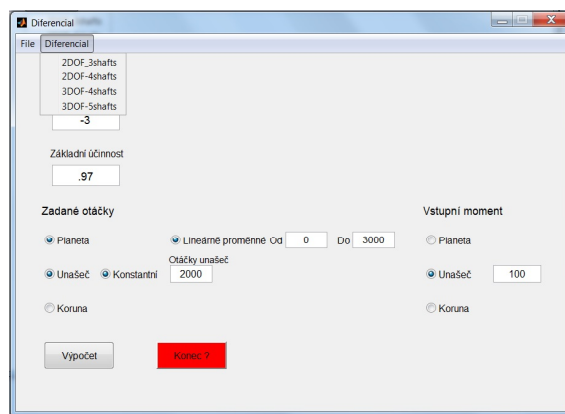


Fig 5: The window for input values of analysis of kinematic and momentary parameters of multiple DOF's differentials.

CONCLUSION

The tool for analysis of kinematic, momentary and energetic parameters of most from the existing mechanisms is accomplished. The separate chapter which can be further programmed are statically indetermined mechanisms and their behaviour (e.g. locked differential and its behaviour during a curve drive).

Separate complex problem, which remains to the future work is to prepare the computational tool for synthesis of planetary and powersplit mechanisms.

LITERATURE

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