A New Chaotic System Based on State Space Energy Feedback

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Abstract - **Multi-wing chaotic attractors are highly complex nonlinear dynamical systems. A new four-scroll chaotic attractor was found by energy feedback controlling method in this paper. Spectral analysis shows that the system in the four-wing chaotic mode has very broad frequency bandwidth, verifying its random nature, and indicating the prospect for engineering applications such as secure communications, biology, etc. Creating a chaotic system with a more complicated topological structure such as a multi-scroll or multi-wing attractor, therefore, becomes a desirable task and sometimes a key issue for many engineering applications.**

Keywords – state energy; nonlinear; error feedback; stabilization; dissipation; chaotic

I. INTRODUCTION

Chaos is a field in mathematics which has found wide application around us. Chaos theory studies the behavior of dynamical systems which are nonlinear, highly initial condition sensitive, having deterministic (rather than probabilistic) underlying rules which every future state of the system must follow. Such systems exhibit aperiodic oscillations in the time series of state variables. It has a large or infinite number of unstable periodic patterns which is commonly termed as order in disorder. Long term prediction is almost impossible due to the sensitive dependence on initial conditions. Though such effect may seem quite unusual but it is however observed in very simple systems, for example, a ball placed at the crest of a hill might roll into different valleys depending on slight difference in the initial position. Most common chaotic phenomenon is observed in case of regular weather prediction. Other application of chaos theory is pervaded in many fields like geology, mathematics, biology, microbiology, computer science, economics, philosophy, politics, population dynamics, psychology, robotics etc. Some real world applications of chaotic time series are computer networks, data encryption, information processing, pattern recognition, economic forecasting, market prediction etc. In this paper it will shown that linear system controlled by a linear controller with energy feedback can generate required types of signals. Depending on parameters of the controller the generated output signal can be sinusoidal, quasi-periodic or even chaotic.

II. THE STATE SPACE ENERGY APPROACH

Let's consider a class of *finite dimensional nonlinear systems* described in the following *form*

$$\dot{x} = A(x)x + Bu$$
$$y = C(x)x \tag{1}$$

where the matrix *C(x),* defining the *output measurement*, is *not a"priori specified,* and the *structure* of matrices *A(x)* and *B* reads

$$A_n(x) = \begin{bmatrix} -\alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ -\alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 \\ 0 & -\alpha_3 & 0 & \ddots & . & . \\ . & . & \ddots & \ddots & \alpha_{n-1} & 0 \\ 0 & 0 & 0 & -\alpha_{n-1} & 0 & \alpha_n \\ 0 & 0 & 0 & 0 & -\alpha_n & 0 \end{bmatrix}; \ B_n = \begin{bmatrix} 1 \\ 0 \\ . \\ . \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

We start with presentation of some *basic ideas* of the *state space energy based* approach [1, 2, 3, 4, 5]. Let $P_0(t)$ denotes the *output dissipation power* of a *zero input causal system* with an *informational output* $y(t)$ defined by:

$$y(t) = \pm\sqrt{P_0(t)}, \ P_0(t) \geq 0 \tag{3}$$

Let *E(t)* denotes the *instantaneous value of the state space energy* (stored in a state vector *x(t)*):

$$E(t) = \int_{t_0}^{\infty} P_0(\tau)d\tau, \ \forall t: \ t = t_0 \tag{4}$$

The *state space energy conservation principle* holds

$$\frac{dE}{dt} = \langle \psi(x), f(x) \rangle = -P_0(t) \tag{5}$$

where ψ is the *gradient vector* of the *state space energy* potential field *E, f* is the *state space velocity vector,* and $\langle ., . \rangle$ denotes the *operation of dual product.*

Because the *choice of origin* and that of the *state space coordinate system is free* we can *define* the gradient *ψ(x)* of the energy *E* in its *most simple form*:

$$\psi(x) = x^T \rightarrow E = \frac{1}{2}\sum_{i=1}^{n} x_i^2 \tag{6}$$

where n is the *order* of the system representation. In some situations it may be useful to consider the integral of $E(x)$ as an additional concept of the *state space hyper-energy J,* which divided by the *length of interval* $T=[t_0,t_1]$, defines a *mean value* of the $E(x)$.

III. THE STATE SPACE ENERGY FEEDBACK

In this part we try to attack the "problem of oscillations" not from the standard *"observation of reality point of view"*, but from the *opposite direction,* i.e. we intend to develop a *consistent approach* to the real-world situations from a *"generation of controlled oscillations point of view."* The objective is to *stabilize the state space energy E(t)* on any *prescribed value E^** by means of a *linear controller,* but under the assumption that *instead* of the *measured output signal y(t),* the *information* about the *actual value of the state space energy E[x(t)]* is assumed to be *available to the controller.*

At least the following *three interpretations* are natural:

• either the *state space energy is continuously measured*, and a standard linear "output error" controller is used, where not the *informational output y(t}*, but the *integrated output dissipation power* is used in *the feedback informational channel* [2], [3],

• or the *actual state vector x(t) is continuously measured* and used in *the feedback informational channel;* then the corresponding actual value of the *state space energy is computed* and *a standard linear "output error" controller* is used,

• or the *actual values of the input u(t) and that of the output y(t) are continuously measured* and used in *the feedback informational channel, including a state reconstructor* [3] and a *state energy error controller.*

The *structure* for *single input-single output system* with nonlinear part of controller is shown in the Fig.1.
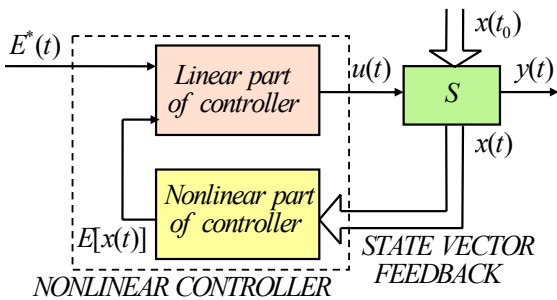

Fig.1. Linear system with nonlinear part of controller and the state vector measurement

Let's start with analysis of the *state representation eq.* (1) where the *information about the system structure* is contained in the triple of matrices *(A, B, C),* e.g. as defined by eq. (2), where the *diagonal elements* of the matrix A represent the *dissipation parameters,* the *off-diagonal elements* of represent the *internal interaction parameters* between both the first order subsystems, and the *elements* of the matrices B and C represent the *parameters of external interactions.* Two typical solutions of the *state energy*

error control problem are illustrated by simulation results: *harmonic oscillations* and *generation of chaotic oscillations.*

The block diagram of linear quadrature oscillator with dissipation is shown in Fig. 2. This system can be described by as second order system with the same dissipation α_1 and frequency proportional of α_2.

$$\begin{aligned} \dot{x}_1 &= -\alpha_1 x_1 + \alpha_2 x_2 \\ x_2 &= -\alpha_2 x_1 - \alpha_1 x_2 \end{aligned} \qquad (7)$$

Because of dissipativity, without control, the oscillation in this system vanishes after short time which depends on initial conditions and values of dissipative coefficient. Using of different controllers is possible for amplitude stabilization of oscillations [6, 7]. In this paper the new principle based on state space energy was selected. The block diagram of linear quadrature oscillator with proportional-integral (PI) controller is displayed.
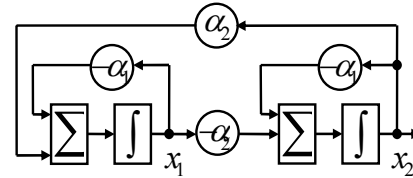

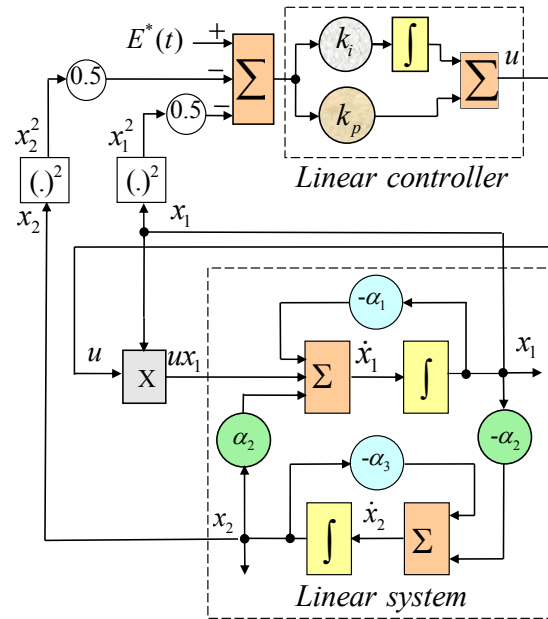Fig. 2. Linear quadrature oscillator with dissipation

Fig. 3. The linear quadrature oscillator controlled with a state energy error PI-controller

The whole system - oscillator with PI controller with nonlinear feedback is represented by

$$\begin{aligned}
\dot{x}_1 &= -\alpha_1 x_1 + \alpha_2 x_2 + x_1 u(t) \\
x_2 &= -\alpha_2 x_1 - \alpha_3 x_2 \\
\dot{x}_3 &= k_I \tilde{E}(t) \\
\tilde{E}(t) &= k_I \left(E^* - E(t) \right) \\
E(t) &= \tfrac{1}{2}\left(x_1^2 + x_2^2 \right) \\
u(t) &= x_1 \left[k_P \left(E^* - \tfrac{1}{2}(x_1^2 + x_2^2) \right) + x_3 \right]
\end{aligned} \qquad (8)$$

The first equation can be rewritten as

$$\dot{x}_1 = x_1(u(t) - \alpha_1) + \alpha_2 x_2 \qquad (9)$$

From this equation can be seen that system can be dissipative if

$$u(t)\text{-}\alpha_1 < 0 \qquad (10)$$

or anti-dissipative if

$$u(t)\text{-}\alpha_1 > 0 \qquad (11)$$

therefore with appropriate control this system can hold desired energy. Based on previous results, the dissipativity/anti-dissipativity is controlled by controller which must hold prescribed energy E^* where state space energy of oscillator is given as

$$E[x(t)] = \frac{1}{2}(x_1^2 + x_2^2) \qquad (12)$$

Results depend on proportional and integral gains of PI controller. It will show that for some gain values of PI controller the system can be chaotic but holds (in average) the prescribed energy. The system can be therefore simply switched as sinusoidal oscillator or system with chaotic oscillations.

IV. RESULTS OF COMPUTER SIMULATIONS

The system according Fig. 3, described by eq. (8) was simulated (System 1) for different values of gains of PI controller. It is important to note that prescribed energy was $E_1^*=1.5$ (from $t=0$ to 150) and $E_2^*=1.4$ (from 150 to 300) with initial conditions $x_1(0)=0.1$; $x_2(0)=0$; $x_3(0)=0$.
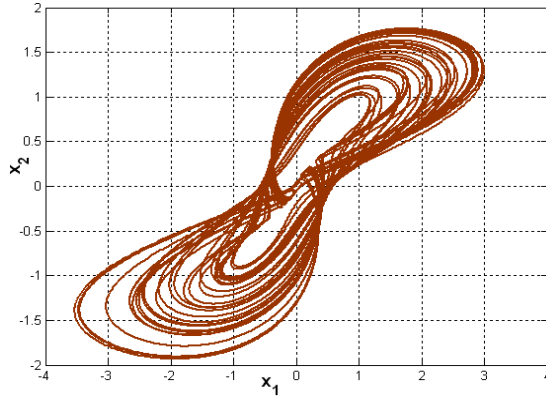

Fig. 4. Phase portrait of system 1 described by eq. (8), for k_i=0.885; k_p=0.099; α_1=1; α_2=1.4; α_3=1.4;
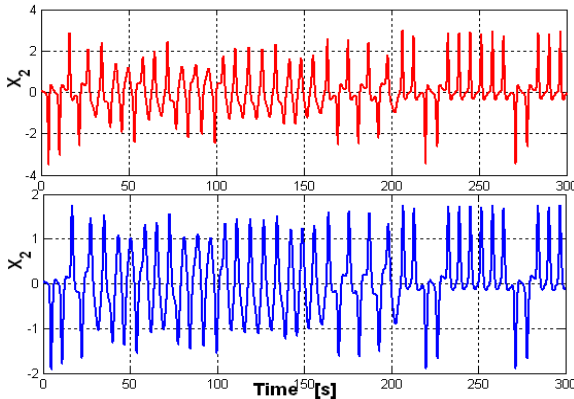

Fig. 5. Time evolution of system 1 described by eq. (8), for k_i=0.885; k_p=0.099; α_1=1; α_2=1.4; α_3=1.4;

The results are shown in following figures: Phase portrait of x_1 and x_2 - Fig. 4, time evolutions of state space variables - Fig. 5, prescribed energy of the control systems (dash line) and true energy (solid line) - Fig. 6. The frequency spectrum of the state space variable x_1 is shown in Fig. 7.
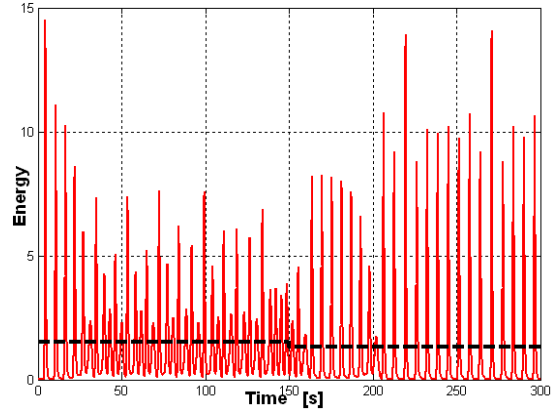

Fig. 6. Time evaluation of energy (system 1) according eq. (12)). Dash line is prescribed energy, solid is state space energy.
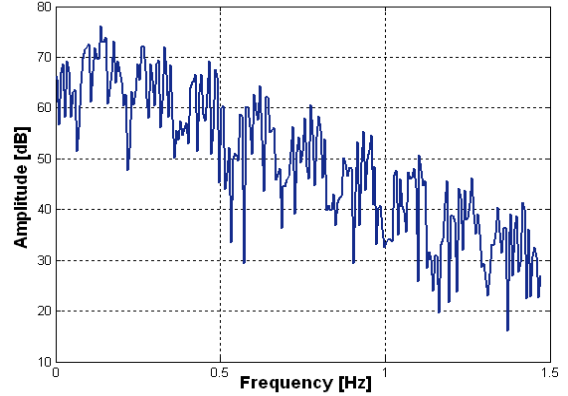

Fig. 7. The frequency spectrum of the state space variable x_1 of system described by eq. (8). System 1

In the next part the same system was used, but the sign of 2 coefficient α_2 and $-\alpha_2$ was exchanged in time $t=150$, therefore "rotation" of the system was reversed. The simulation results leads to 4 wing chaotic system (System 2), see phase portrait in Fig. 8 and the next figures 9, 10, 11, time evolution of state space variables, state space energy and frequency spectrum respectively [8 - 11]. Initial conditions are $x_1(0)=0.1$; $x_2(0)=0.01$; $x_3(0)=0$
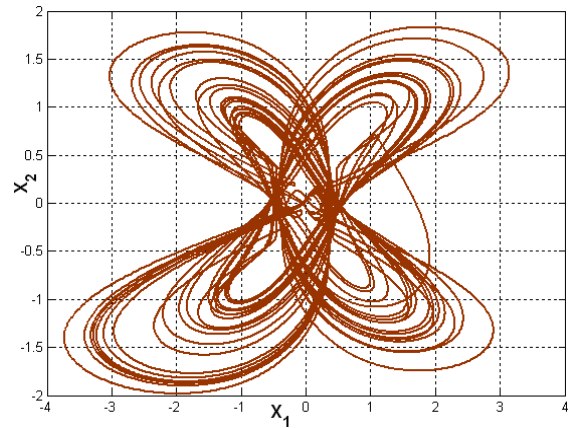

Fig. 8. Phase portrait of system 2 with reversed rotation, k_i=0.885; k_p=0.099; α_1=1; α_2=1.4; α_3=1.4; E_1=1.50; E_2=1.4
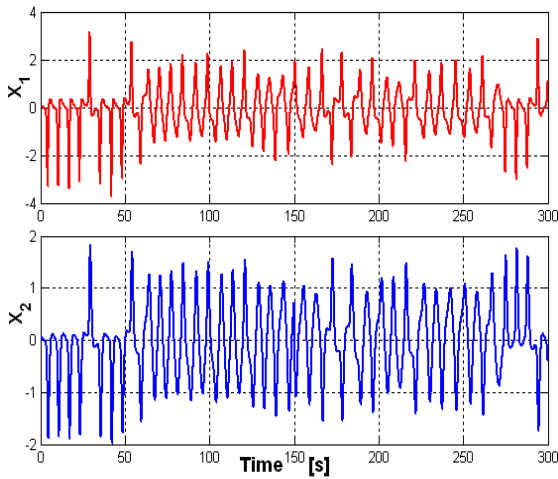
Fig. 9. Time evolution of system 2 with reversed rotation, k_i=0.885; k_p=0.099; α_1=1; α_2=1.4; α_3=1.4; E_1=1.50; E_2=1.4
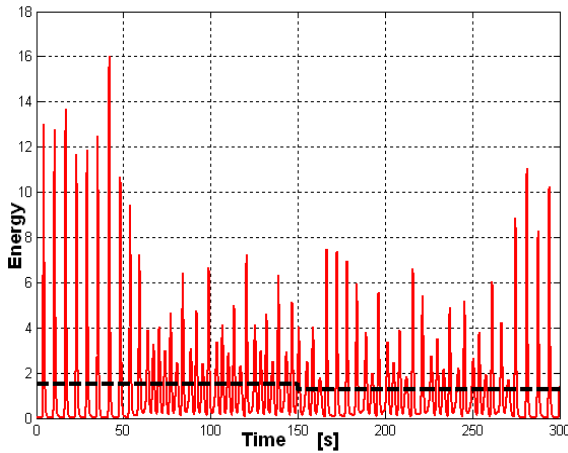

Fig. 10. Energy of system 2 with reversed rotation, k_i=0.885; k_p=0.099; α_1=1; α_2=1.4; α_3=1.4; E_1=1.50; E_2=1.4
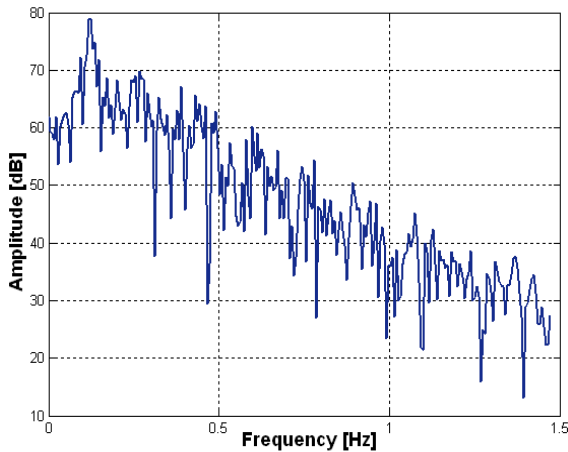

Fig. 11.The frequency spectrum of system 2 with reversed rotation, k_i=0.885; k_p=0.099; α_1=1; α_2=1.4; α_3=1.4; E_1=1.50; E_2=1.4

TABLE I.   SYSTEMS PROPERTIES.

	System 1		System 2	
	x1	**x2**	**x1**	**x2**
Lyap. Exponent	0.031	0.037	0.038	0.048
Hurst exponent	0.71	0.798	0.713	0.80
Capacity Dim.	1.32	1.29	1.36	1.27
Correl. Dim.	1.67	1.71	1.66	1.56

In Tab. I some important characteristic of the booth systems are presented – Largest Lyapunov exponent, Hurst exponent, Capacity dimension and Correlation dimension.

CONCLUSION

In this paper the chaotic system based on linear system, linear controller but nonlinear state space feedback was presented and simulated. The theory of state space energy approach was used. Depending on parameters of the controller the system can generate sinusoidal, quasi-periodic or even chaotic signal.

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