TOWARDS ALTERNATIVE CONFIGURATIONS TO DETERMINE THE TENSILE STRENGTH OF BRITTLE MATERIALS

Christos F. MARKIDES[1], Stavros K. KOURKOULIS[1]

[1] National Technical University of Athens, Department of Mechanics, Laboratory for Testing and Materials, 5 Heroes of Polytechnion Avenue, Theocaris Building., Zografou Campus, 157 73, Athens, Greece, E-mail: markidih@mail.ntua.gr; stakkour@central.ntua.gr

1. Introduction

An alternative configuration of the ring test [1], which was introduced as a convenient substitute of the familiar Brazilian-disc test [2] for the determination of the tensile strength of brittle materials, is described here. The configuration proposed is complementary to the one discussed in a recent paper [3], dealing with the flexure of curved beams and the uniaxial stress state developed at critical points of its circumference, leading to tensile fracture.

The main features of the configuration are shown in Fig. 1. It consists of circular ring with a diametral cut, loaded by forces, *P*, acting normally to the cut and directed outwards with respect to the cut. Based on the well-known solutions for the curved beam, provided by Muskhelishvili [4], the maximum tensile stress, developed at the inner circumference of the ring at point *B*, σ_θ^B, (Fig. 1) is calculated in terms of geometry and *P*. The main idea is that σ_θ^B represents the tensile strength of the material, assuming that *P* attains the critical value that causes fracture.

According to Saint-Venant's principle, the stress field developed along the critical locus *AB* is due to

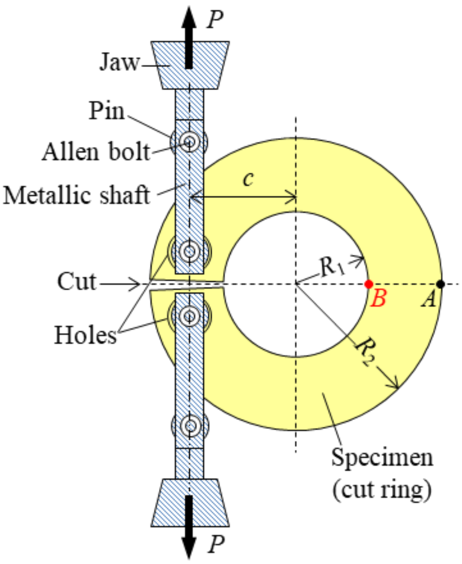

Fig. 1. The configuration of the problem.

two factors: (i) couples of magnitude *Pc*, where *c* is the eccentricity (Fig. 1), and (ii) a pair of collinear point forces, *P*, acting on the edges of the Circular Semi Ring (CSR) (Fig. 2). Both cause "opening" of the curved beam. Due to the high eccentricity, the couples induced are of high magnitude, resulting to fracture at point *B* under relatively low *P*-values.

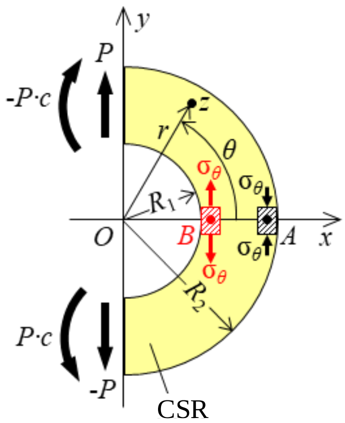

Fig. 2. The mathematical problem for the CSR.

2. Theoretical considerations

The stress field (and thus the tensile strength of the material) is here obtained in closed form using Muskhelishvili's formulae for the curved beam [4], assuming that the configurations of Figs. 1 and 2 are equivalent (at least for points relatively far from the load application area, as it is the neighborhood of line *AB*, where the maximum and minimum principal stresses are developed). In turn, assuming linear elasticity, the solution for the CSR is obtained by superposing the solutions of two problems, which are provided by Muskhelishvili, in terms of the dislocations concept [4]: Problem I, which deals with the CSR under couples $M=Pc$ (acting on its straight edges) and Problem II, dealing with the CSR under shear forces, *P* (both acting on its straight edges).

In this context, assuming that the CSR is lying in the complex plane $z=re^{i\theta}$ ($-\pi/2\leq\theta\leq\pi/2$, $R_1\leq r\leq R_2$), Fig. 2, with $\rho=R_2/R_1$, the relevant stress fields due to these two problems (I, II), read separately as:

$$\sigma_r^{(I)} = \frac{-2Pc}{hR_1^2} \cdot \frac{\left(\rho^2-1\right)\log\frac{r}{R_1} + \rho^2\left(\frac{R_1^2}{r^2}-1\right)\log\rho}{\left(\rho^2-1\right)^2 - 4\rho^2\left(\log\rho\right)^2} \quad (1)$$

$$\sigma_\theta^{(I)} = \frac{-2Pc}{hR_1^2} \cdot \frac{\left(\rho^2-1\right)\left(\log\frac{r}{R_1}+1\right) - \rho^2\left(\frac{R_1^2}{r^2}+1\right)\log\rho}{\left(\rho^2-1\right)^2 - 4\rho^2\left(\log\rho\right)^2} \quad (2)$$

$$\tau_{r\theta}^{(I)} = 0 \quad (3)$$

$$\sigma_r^{(II)} = \frac{P}{2h} \cdot \frac{\frac{\rho^2+1}{r} - \frac{r}{R_1^2} - \frac{\rho^2 R_1^2}{r^3}}{\left(\rho^2+1\right)\log\rho - \rho^2 + 1} \cdot \cos\theta \quad (4)$$

$$\sigma_\theta^{(II)} = \frac{P}{2h} \cdot \frac{\frac{\rho^2+1}{r} - \frac{3r}{R_1^2} + \frac{\rho^2 R_1^2}{r^3}}{\left(\rho^2+1\right)\log\rho - \rho^2 + 1} \cdot \cos\theta \quad (5)$$

$$\tau_{r\theta}^{(II)} = \frac{P}{2h} \cdot \frac{\frac{\rho^2+1}{r} - \frac{r}{R_1^2} - \frac{\rho^2 R_1^2}{r^3}}{\left(\rho^2+1\right)\log\rho - \rho^2 + 1} \cdot \sin\theta \quad (6)$$

Summing up Eqs. (2) and (5), σ_θ is obtained for the problem shown in Fig. 2. Clearly, at point $B(R_1,0)$, the only non-zero stress component is the maximum principal stress $\sigma_\theta^B>0$, while at point $A(R_2,0)$ the only non-zero stress component is the minimum principal stress $\sigma_\theta^A<0$. It is seen that the ratio $k=\sigma_\theta^B/|\sigma_\theta^A|$

$$k = \left\{2c\rho\left[\rho^2 - 1 - \left(3\rho^2+1\right)\log\rho + 2\rho^2\frac{\rho^2+1}{\rho^2-1}\left(\log\rho\right)^2\right] + R_2\left[\left(\rho^2-1\right)^2 - 4\rho^2\left(\log\rho\right)^2\right]\right\} \Big/ \left\{2c\rho\left[1-\rho^2+\left(3+\rho^2\right)\log\rho - 2\frac{\rho^2+1}{\rho^2-1}\left(\log\rho\right)^2\right] + R_1\left[\left(\rho^2-1\right)^2 - 4\rho^2\left(\log\rho\right)^2\right]\right\} \quad (7)$$

always exceeds unity. It is therefore concluded that brittle materials of compressive strength significantly exceeding the tensile one, will likelihood fail, due to the configuration of Figs. 1 and 2, at point B, under a uniaxial tensile stress field. The ratio k is independent of the magnitude of the external load P, depending exclusively on the dimensions of the cut ring and the eccentricity c. Assuming R_2=5 cm, and $c=(R_1+R_2)/2$, the variation of k against the radius R_1 of the inner hole is plotted in Fig. 3.

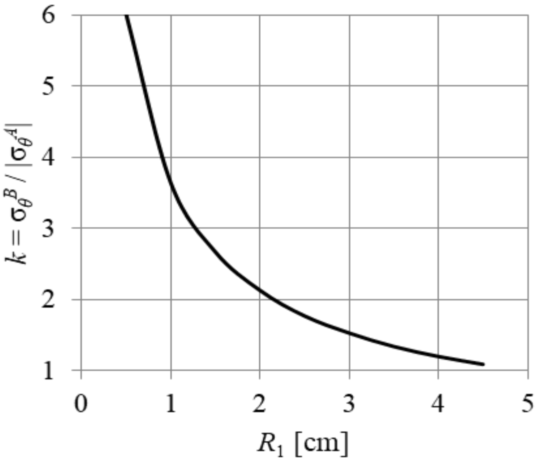

Fig. 3. Ratio k versus inner hole's radius R_1.

3. Discussion and Conclusions

An alternative procedure for the determination of the tensile strength of brittle materials by means of the cut-ring specimen, in conjunction with proper closed-form analytic formulae based on Muskhelishvili's solutions for the respective CSR configuration, was proposed. It was concluded that the tensile strength may be effectively determined in the case of a moderate inner hole of the ring.

It must be noticed however, that the proposed configuration should be handled with caution. For high R_1-values secondary effects due to buckling could influence the experimental procedure. On the other hand, for small R_1-values, Saint-Venant's principle should also be checked regarding the transition from the configuration of Fig. 1 to that of Fig. 2.

The procedure proposed is also to be tested for possible description of the stress concentration in the case of a small inner hole, somehow resembling relevant methods of ASTM standards [5].

References

[1] Ripperger, E., Davis, N. Critical stresses in a circular ring. *Trans. Am. Soc. Civil Engrs*, 1947, 619–627.
[2] Akazawa, T. New test method for evaluating internal stress due to compression of concrete. *J. Jpn Soc. Civil Engineers*, 1943, 29, 777-787.
[3] Kourkoulis, S.K., Pasiou, E.D., Markides, C.F. Analytical and numerical study of the stress field in a circular semi-ring under combined diametral compression and bending. *Frattura ed Integrità Strutturale*, 2019, 47, 247–265.
[4] Muskhelishvili, N.I. *Some basic problems of the mathematical theory of elasticity*, Noordhoff, Groningen, 1963.
[5] ASTM E399 - 90. *Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials*, 1997.