Research Article

Statistical approaches on the design of fatigue stress spectra for bus structures

Rando Tungga Dewa[1] · Miloslav Kepka Jr.[1] · Miloslav Kepka[1]

Abstract

This paper presents technical procedures and statistical approaches on the extrapolation of fatigue stress spectra for the critical part of the bus structure. The approach for such stress spectra extrapolation can be used for determining long-term stress spectra at expected service life. Some extrapolation concepts are presented to show the suitable determination of the experimental data. The conventional Non-parametric and the simplified extreme value extrapolation approaches are utilised in the present case. The mathematical model is then used to verify the anticipated extreme stress level to some extrapolation factor, which is almost coincide. As a result, the extrapolated stress spectra histograms are compared with the evaluated fatigue damage and cycles counted based on several improved criteria. The extrapolated histograms appear to be reasonable results with higher stress amplitudes compared to the measured one and greatly contribute to the cumulative damage. This study results in satisfactory findings to the best of knowledge on the studied case. Thus, certain techniques and best practices are included in this article to be followed.

Keywords Stress spectra · nCode Glyphworks · Monte Carlo simulation (MCS) · Extreme value extrapolation (EVE) · Non-parametric (NP) · Statistical analysis

1 Introduction

For years, Regional Technological Institute (a research centre affiliated with the Faculty of Mechanical Engineering of the University of West Bohemia, Pilsen, Czech Republic) has been contributing and developing an advanced methodology for testing structures of vehicle components for our customers [1–3]. Typical complex and repeated random loadings are obvious in vehicle structure during service operation. It is necessary to perform fatigue analysis and reliability assessment of the structure based on stress spectra analysis as a representative for the service condition. Stress spectra is a strong tool to characterise random loadings from time-series measurement. However, it is not possible to measure practically the complete time-series information based on the design life requirement owing to some limitations in testing capacity, time, and cost. Therefore, such a stress extrapolation is essential to determine an accurate long-term time-series from a measured one.

The conventional method for stress expansion has been done by using the superposition principle. Since this method is based on data repetition, it was failed to capture the extreme stresses that have a large impact on fatigue damage of components [4]. Over the past years, researchers have proposed several stress extrapolation methods [5–8]. Socie [6] reported a methodology for estimating the long-term durability of structures from short-term service loads measurements via non-parametric (NP) statistical approach. The kernel density (KD) was employed for the NP density estimation to extrapolate the short-term measured usage. Johannesson [7] performed the extreme value extrapolation (EVE) analysis in the time domain based on the peak-over-threshold (POT) extraction method.

✉ Rando Tungga Dewa, rando.td@gmail.com; rando@rti.zcu.cz | [1]Regional Technological Institute (RTI), Research Centre of Faculty of Mechanical Engineering, University of West Bohemia, 301 00 Pilsen, Czech Republic.

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Furthermore, some methods of improvement were proposed by Cerrini et al. [8] to take into account a bigger and more damaging cycles and smaller cycles caused by two different service operations in the time and rainflow domain.

In order to predict the most accurate design life, selecting a suitable stress extrapolation concept is of great concern according to the characteristics of stress and the fatigue strength. The relationship between deterministic and probabilistic solutions in fatigue random loading can be very useful to overcome the upcoming issues. In this paper, the NP approach from direct computing method of a commercial software nCode Glyphworks [9] and EVE statistical analysis from simplified algorithm are utilised for extrapolation of a time-series to a longer period. This study elaborates in-depth interpretation of the measured time-series from a bus in real operation at Czech Republic. Monte Carlo sampling (MCS), to provide denser distribution of stress amplitude will first perform a modelling of the gauss distribution for stress spectra and some reliability values of extreme stresses can be obtained. The NP extrapolation is conducted via computing analysis, yet the signal noises will first filter out from the time series. The simplified method for EVE analysis is then introduced, and the Excel programming somehow can perform the method. For the case of study only, the present paper uses a number of tenfolds extrapolation for the sake of time saving and accuracy. Finally, all results will be compared in terms of stress spectra histogram and their fatigue damages.

2 Data acquisition and pre-processing

The most accurate representative record of the stress-time history is from the service time-series measurement. The stochastic process of measured stress are transformed into the one- or two-parameter stress spectra histogram by the widely used rainflow counting method. This method and its algorithm are presented in Refs. [10, 11]. This one- and two-parameter stress spectra histogram is required for calculating the fatigue damage and life prediction of the structures. The time-series measurement of an actual bus (passenger-loaded) was measured through an equipment system with complex strain gauges attached in some critical structures of the bus along the irregular city track in (Myšlínská-Koloděje) Czech Republic with approximately 11 km in distance; therefore, some manoeuvres (curve riding, braking, typical driver, etc.) are expected. The measured frequency was varied about 100–200 Hz. In this case, the most critical part was selected, which is located in the top corner of the door in the bus body structure, as shown in Fig. 1a. The structure was manufactured from the type of X_2CrNi_{12} stainless steel for parent material.

Due to some uncertainties from external factors, signal noises such as insignificant peaks, distortions, and spikes were firstly filtered out before rainflow processing. During the rainflow counting process, small cycles with amplitudes below 2 MPa that do not contribute to the fatigue damage were eliminated from the calculated stress spectra. The results of time-series and one-parameter stress spectra histogram can be seen in Fig. 1b.

3 Modelling of stress spectra

3.1 Gauss distribution

As above-mentioned, the fatigue random loading is linked to the stochastic process in it. Since the random stresses can be regarded as a continuous variable, the occurrence of stress over time in randomness of fatigue can be modelled through the famous gauss distribution, which is following a normal distribution. Its probability density function (PDF) can be used as a function of stress occurrence at any given time on the specific bins. The PDF of normal distribution is expressed as:

$$f(x) = \frac{1}{std\sqrt{2\pi}} e^{-\frac{(x-mean)^2}{\left(2 x\, std^2\right)}} \tag{1}$$

where *mean* and *std* stand for the mean or expectation of the distribution and standard deviation respectively.

3.2 Monte Carlo sampling

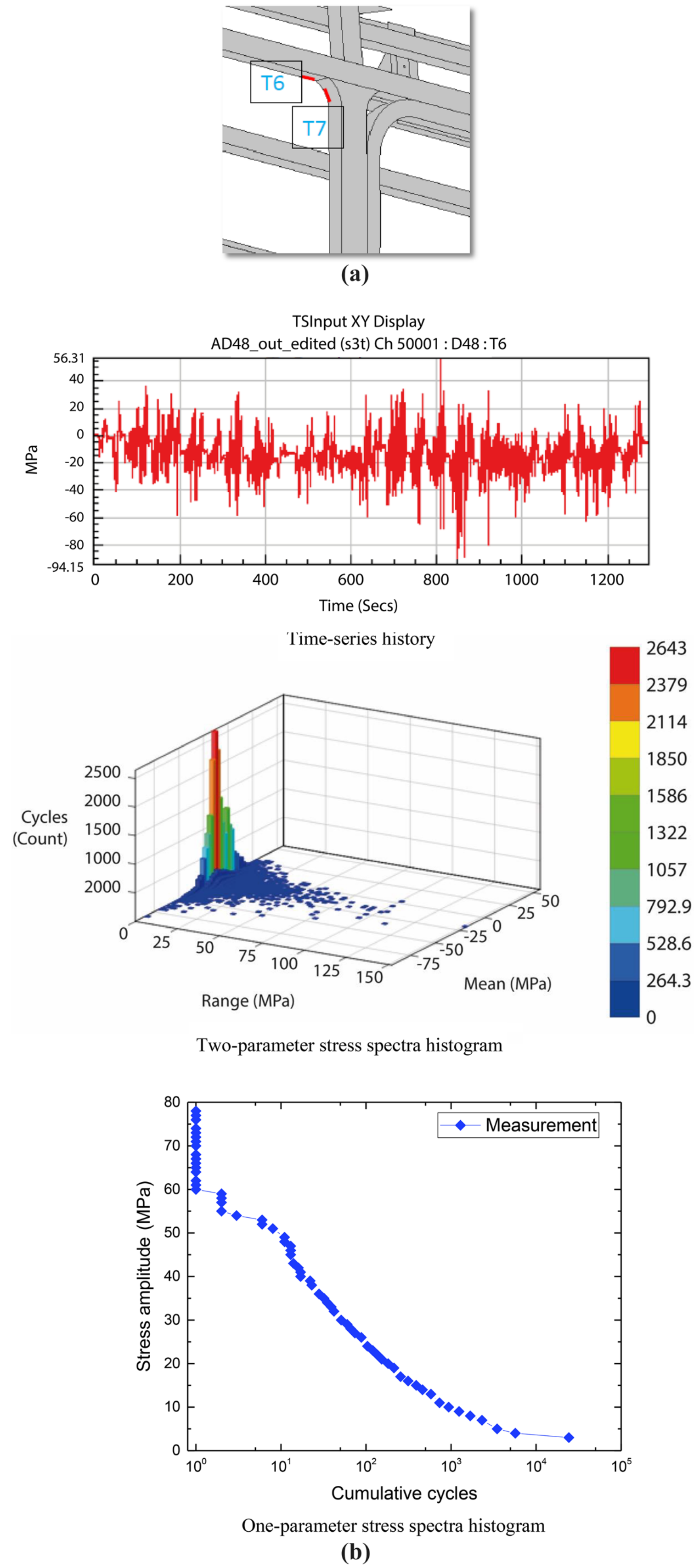
In this section, the MCS technique is adapted to provide a denser PDF of stress amplitude. For problems involving random variables, the MCS technique is found to provide useful information. The process involves a generation of set of values with corresponding probability distribution, thus, a sample of solution can be derived. In our case, the MCS is performed to generate random variables for two random parameters (x_1 and x_2) with uniform PDF between 0 and 1. Box–Muller equation can be used to represent the random variable (U) with a standard normal distribution as follow:

$$U = \left(-2\ln x_1\right)^{0.5} \cos 2\pi x_2 \tag{2}$$

Thus, the normal distribution of random variables (y_i) can be given as:

$$y_i = mean + std[U] \tag{3}$$

Fig. 1 **a** Schematic of illustration for strain gauge location at critical structure, **b** time-series and stress spectra histogram from experimental measurement



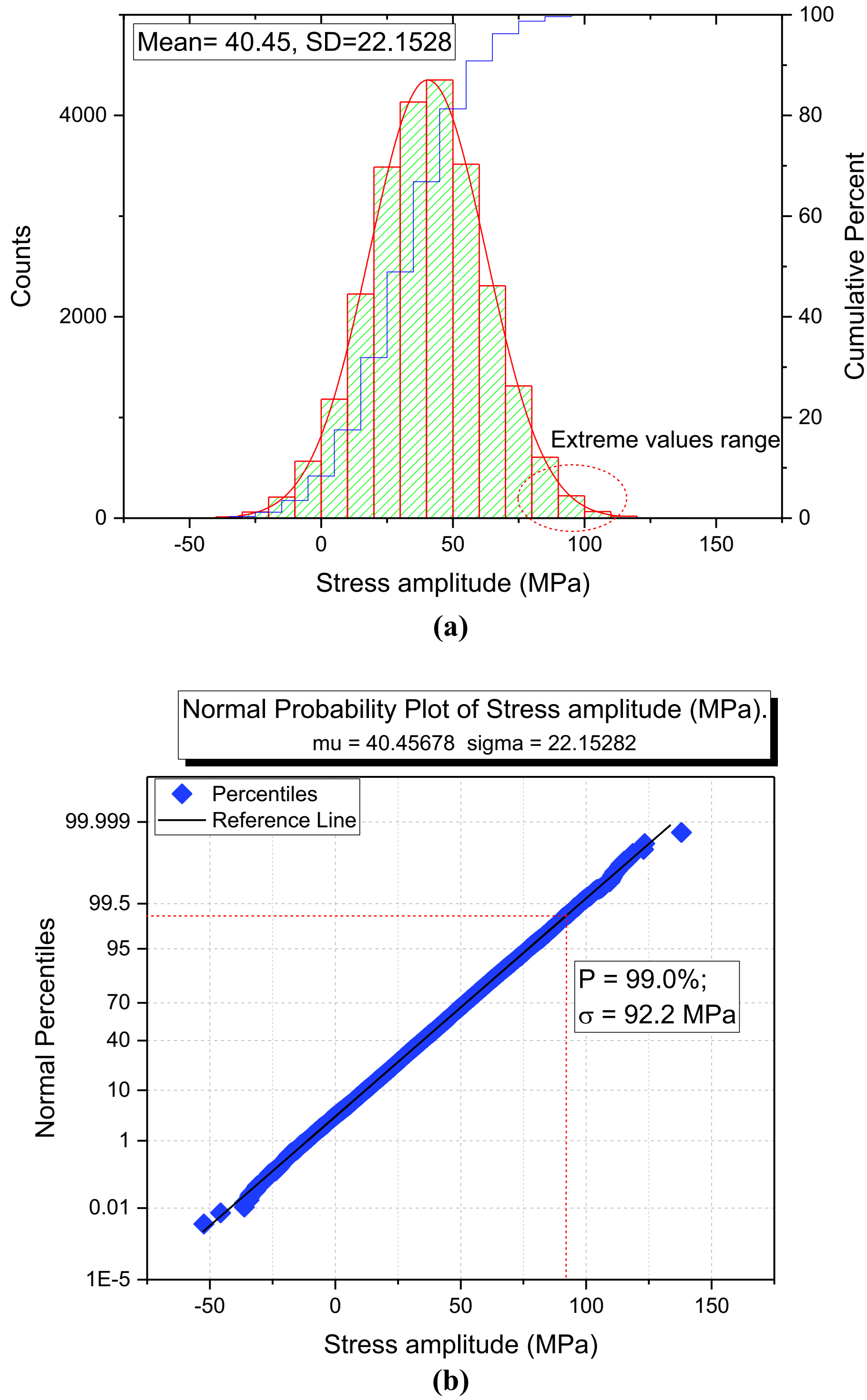


Fig. 2 **a** The PDF of large amount of stress amplitude obtained by MCS, **b** the probability plot of stress amplitude distribution

Equation (3) is used to generate x_1 and x_2, which follows a normal distribution. Figure 2 shows the PDF for MCS results of the generated 25,000 data points of stress amplitude. From the figure, it is obvious that the stress amplitudes distribution follows a normal distribution because the probability plot shows a linear agreement with all data. According to Fig. 2, the design of extreme stress amplitude for some reliability values can be obtained. Table 1 shows a summary of results for some stress amplitudes that can be used as references for the probability occurrence in case of stress extrapolation. For example, 100-folds of extrapolation from the measured stress spectra will result in at least the extreme stress amplitude as equal to 110 MPa, or the probability

Table 1 Design of extreme stress amplitudes for some reliability values

Reliability (Probability of occurrence) (%)	Design of extreme stress ranges (MPa)
90.0 (10.0)	68.8
95.0 (5.0)	76.9
97.7 (2.3)	85
99.0 (1.0)	92.2
99.9 (0.1)	110
99.99 (0.01)	120.9
99.999 (0.001)	133.6

of extreme stress amplitude that will occur can also be as high as 133.6 MPa.

3.3 Non-parametric extrapolation

The NP approach can be utilised in the statistical population of rainflow matrix (RFM). In principle, this method of extrapolation can be described as using the NP approach to generate the statistical probability distribution of the RFM. As a result, this method can be conservative because it requires no assumptions of the data distribution, or simply the characteristic of time series data. In addition, the KD is applied to represent the impact of surrounding of the bins or can be said, more cycles will be placed in the bins with higher density (determined by KD). Hence, the result can be thought as fitting the probability distribution of the histogram. In this paper, the commercial software, namely nCode Glyphworks, is used to calculate the direct method of computing the extrapolated RFM. In software nCode, the procedure is simply extrapolating the From-To RFM to longer periods by means of extrapolation factor (*e*). Figure 3 shows the density and two-parameter stress spectra histogram from tenfolds extrapolation.

3.4 Extreme values extrapolation

Fundamental rule based on extreme values extrapolation is to obtain the extreme stresses not observed in the short-time measurement. The procedures are simplified as from the literature such as rainflow filtered and extraction of turning points (excesses) from time-series, modelling the distribution of excesses, generating a new random distribution of excesses to be placed in the new time-series block, repeating the process until *n*-fold, and finally, the

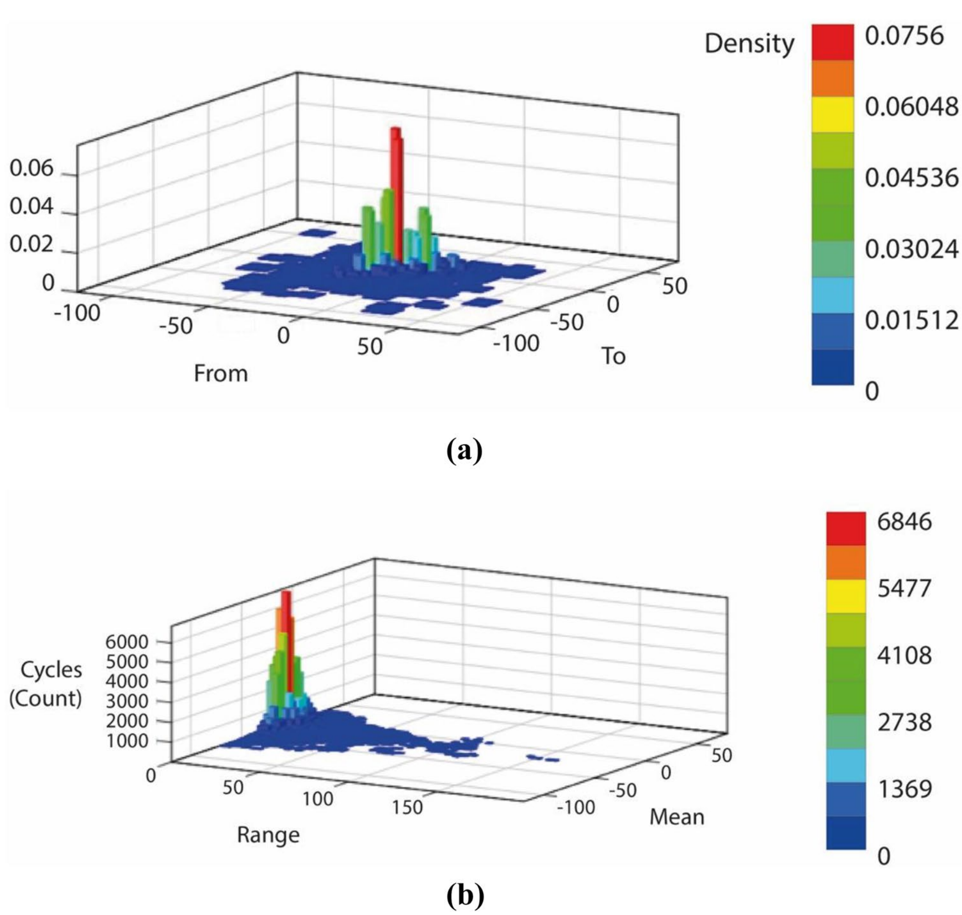


Fig. 3 **a** The density histogram, **b** the two-parameter stress spectra histogram of the tenfolds extrapolation by the NP statistical method

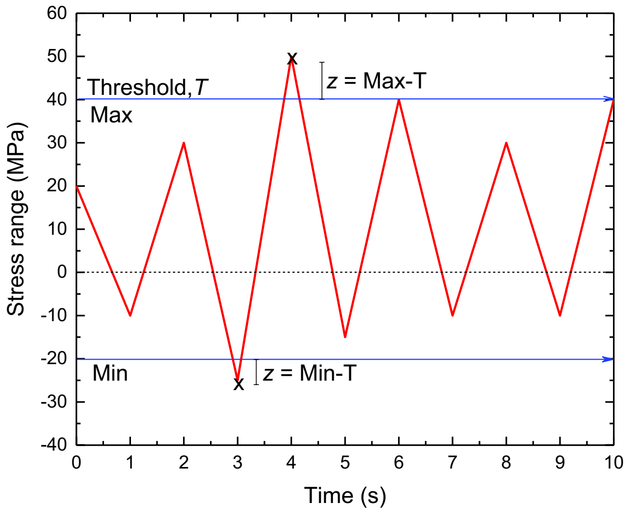


Fig. 4 Schematic illustration of POT method

n-fold extrapolation can be obtained by placing repeatedly the new time-series block one after one.

Based on extreme values statistic, the theoretical base of turning points extraction in our work is called the peak-over-threshold (POT) method, which is schematically shown in Fig. 4. It is important to note that the determination of threshold level is highly decisive. The value must be high enough and low enough to produce a good distribution. The rule of thumb can be adapted as the number of exceedances are about the number of square roots of the total cycle in time series. The excesses, *z*, above the threshold can be modelled using the exponential distribution under certain conditions [$F(z) = 1 - \exp(z - \text{mean})$]. In this study, the excesses over the maximum threshold of 20 MPa and below −50 MPa are plotted in Fig. 5. The exponential probability plots show that the distribution of excesses are in good agreement, although a few data is seen out of the straight line that can be eliminated (red circle). As a

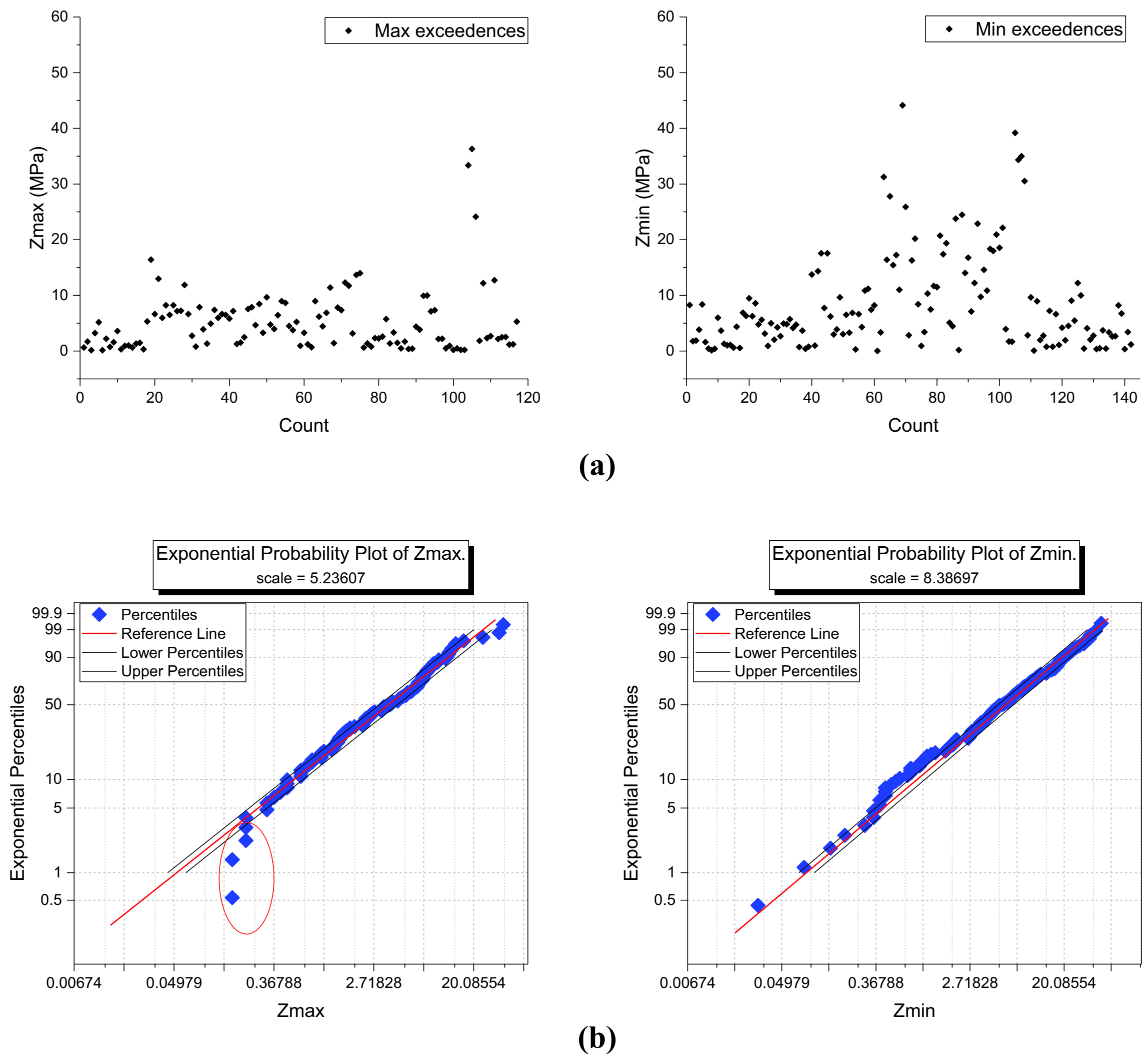
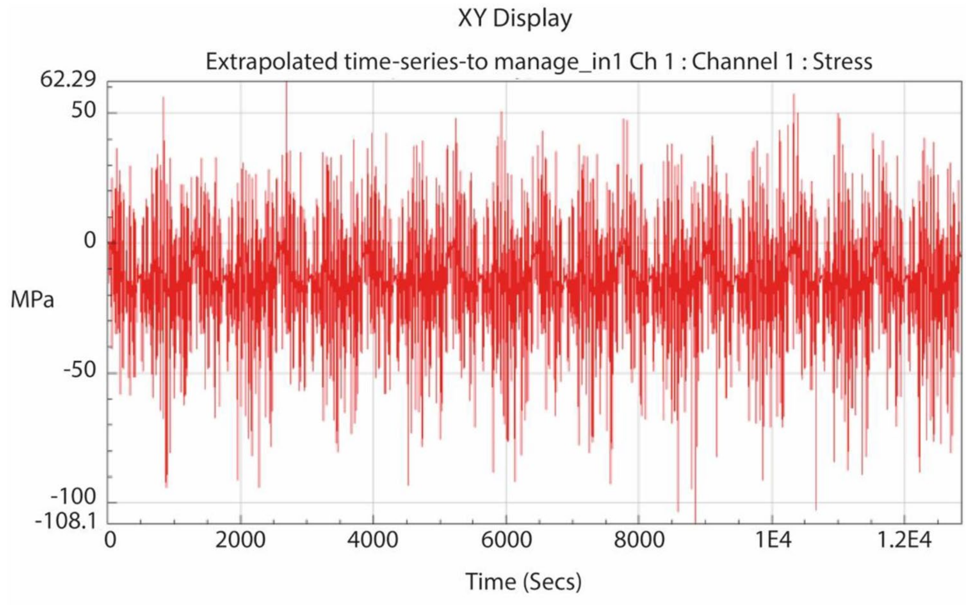
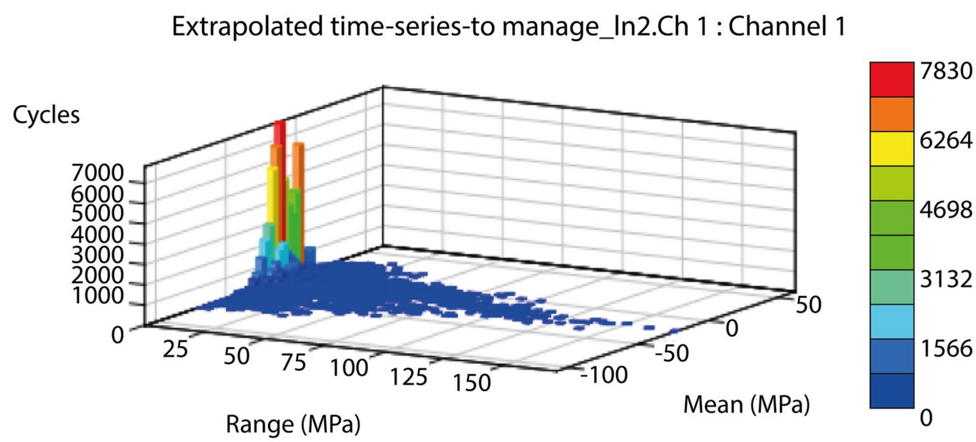


Fig. 5 **a** Points of exceedances extracted from time-series, **b** the exponential probability plot of exceedances

Fig. 6 **a** The tenfolds extrapolated time-series, **b** the two-parameter stress spectra histogram of the tenfolds extrapolation by the EVE method



(a)



(b)

result, Fig. 6 shows the tenfolds extrapolated time-series and two-parameter stress spectra histogram.

4 Fatigue damage evaluation

Based on technical experience, the welded structure of the bus is a uniaxially T-joint loaded in their normal stresses with respect to its cross sections and the high-cycle fatigue uniaxial approach is sufficient in the present case. The fatigue S–N curve is estimated based on empirical formulas or material database and standards containing design fatigue curves for various geometric and design configurations of structural details. The British Standard BS 7608 [12] related to the design and assessment of steel structures is used in this study for design parameter. This standard includes T-class fatigue curve estimation according to the structural hot-spot stress. The smaller stress ranges below the fatigue strength are included in the analysis by using the Haibach approach, in order to take into account some damage increments from small stresses. Fatigue life and damage evaluation are performed via nCode software. Thus, all parameters are converted into the format used in the software. The S–N curve function is expressed as follow:

$$\Delta\sigma = 5071.66 \cdot (N_f)^{-0.2} \tag{4}$$

where $\Delta\sigma$ and N_f are stress range in fatigue constant amplitude loading and number of cycles to failure. The critical life, N_c, of 10^6 cycles is chosen for estimation.

The linear cumulative damage, *D*, rule is adapted in this work to estimate the fatigue damage in random loading with respect to the fatigue constant loading data. According to the concept, the fatigue limit (failure) state is reached when the following condition is satisfied:

$$\sum_i \frac{n_i}{N_i} = D \geq 1 \tag{5}$$

where n_i is the number of cycles for the case of fatigue random loading and N_i is number of cycles to failure for the case of fatigue constant loading (obtained from the selected S–N curve) at the same stress level.

5 Results and analysis

From mathematics understanding, Hinkelmann et al. [13] contributed to the estimation of maximum stress amplitude through numerical derivation and calculation. This method can be used only for predicting the maximum stress amplitude and when it is combined with the superpositioned stress-spectra histogram, and the reconstructed one-parameter stress spectra histogram after some extrapolation can be obtained. To begin, the predicted life, H, at certain stress amplitude (S_h) of spectra can be described as:

$$H = H_i^{1-\left(\frac{S_h}{S_i}\right)^s} \tag{6}$$

The extrapolation factor is represented with e. Thus, the ratio of service life and duration of measurement can be shown as:

$$e = \frac{H_i}{H_0} = H_i^{1-\left(\frac{S_0}{S_i}\right)^s} \tag{7}$$

where H_i and H_0 are extrapolated cumulative cycle and measured cumulative cycle, respectively. S_i and S_0 are extrapolated stress amplitude and measured stress amplitude, respectively. Finally, s denotes the typical stress spectra shape. If we substitute to the exact Eq. (7), thus, we have:

$$e = H_i^{1-\left(\frac{S_0}{S_i}\right)^s} \rightarrow \frac{H_i}{H_0} = H_i^{1-\left(\frac{S_0}{S_i}\right)^s} \rightarrow \frac{1}{H_0} = H_i^{-\left(\frac{S_0}{S_i}\right)^s} \tag{8}$$

We further logarithmising and transforming both sides of Eq. (8). Hence, the predicted maximum stress range based on the quantity of e can be identified through:

$$\log\left(H_0\right) = \left(\frac{S_0}{S_i}\right)^s \log\left(H_i\right) \tag{9}$$

$$\frac{S_i}{S_0} = \left[\frac{\log\left(H_i\right)}{\log\left(H_0\right)}\right]^{1/s} \tag{10}$$

$$S_i = S_0\left[\frac{\log\left(H_i\right)}{\log\left(H_0\right)}\right]^{1/s} \tag{11}$$

In the previous work [2], the shape characteristic is categorised into three shapes of stress spectra based on the cumulative frequency in semi-logarithmic scale. In our study, $s=2$ is chosen to take into account the manoeuvres (curve riding, braking, etc.) during operation, thus lead to stress spectra with normal distribution. Figure 7 shows some results for tenfolds and 100-folds calculation from Eqs. (6–11) and compares with the measured stress spectra. It is clear that the calculated maximum stress amplitude is obtained to a higher amount of stress amplitude with increasing of the e.

To see if all the extrapolation results can be satisfying, Fig. 8 shows all results of one-parameter stress spectra after tenfolds extrapolation. From the figure, it is observed that the extrapolation of histograms appears to be reasonable results with higher stress amplitudes compared to the measured ones. For tenfolds extrapolation case, the maximum stress amplitude obtained by the simplified-EVE and NP methods is 87.8 and 82.8 MPa, respectively. It is verified through mathematical derivation, which equals to 86.5 MPa and the probability of occurrence for tenfolds extrapolation can reach a maximum of 92.2 MPa based on MCS technique. The authors expect this type of result. Furthermore, the fatigue damage calculation has been performed, and Table 2 summarises the results. It is clear that both EVE and NP methods show a higher fatigue damage compared to the superposition technique, evidencing the maximum

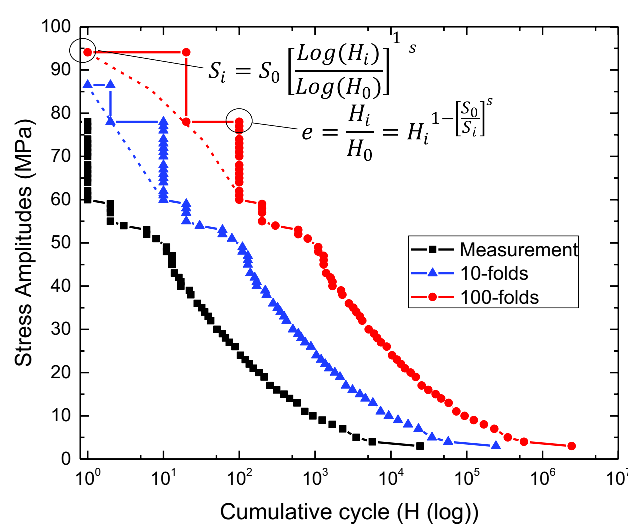


Fig. 7 The reconstructed of tenfolds and 100-folds one-parameter stress spectra calculation and compared with the measured one

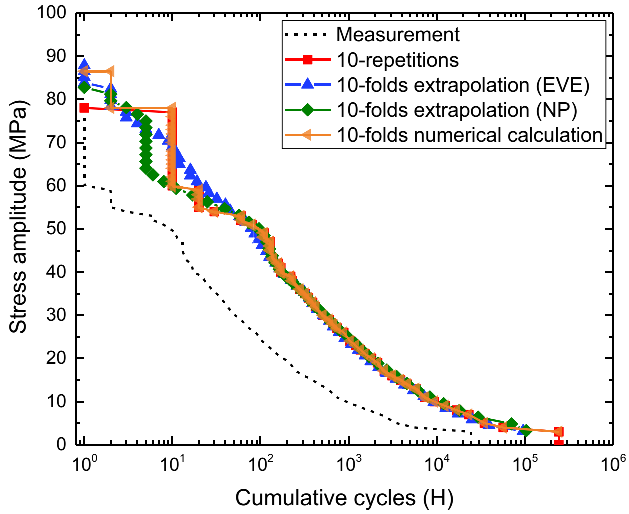


Fig. 8 Comparison of one-parameter stress spectra after tenfolds extrapolation with the studied method of extrapolation, respectively

Table 2 Comparison of the extrapolated cumulative damage estimation by superposition, NP, and simplified-EVE techniques, respectively

Methods	Number of cycles counted	ΣD
Superposition	2.43E5	1.92E−6
Non-parametric (NP)	1.04E5	5.81E−6
Extreme values extrapolation (EVE)	2.37E5	6.59E−6

stress amplitude greatly contributes to the cumulative damage. However, the number of cycles counted from the NP technique has the lowest value because of cycles concentration occurs at the small stress amplitudes. In conclusion, all methods can be utilised in the present case for bus component with obtained characteristic of stress-time history. Each method has their advantages and disadvantages, EVE method is able to produce a direct extrapolated time-series history, if needed. The generation of cycle sequence is indeed more complex but resulting in more reasonable stress amplitude. For the case of thousand extrapolations, it is recommended to use a special computing software for statistical extrapolation. The NP technique is relied on the type and size of the RFM. It provides multi-variate distribution of the RFM, thus, the result can be conservative (meaning that the obtained stress amplitude can be over- or under- estimated) on occasion. This technique is best to use if there is a small dependence on the distribution of the RFM. However, the simulated stress amplitudes on each technique are randomly placed on the histogram bins. Therefore, to predict a random event is obvious to require uncertainties involved, thus, the extrapolated histogram can be unique and different histogram will be produced in each calculation.

6 Conclusions

In this study, we have identified the technical procedures and statistical approaches of response stress spectra for actual measurement of passenger-loaded bus structures. Measured spectra was a true representative of real service loads and easy to analyse. Stress-time histories data filter was applied to reduce the amount of unnecessary data and revealing only significant and have huge impact in fatigue life. Some statistical approaches were investigated and simplified at desired extrapolation factor to satisfy the longer stress spectra. The NP and simplified-EVE approaches were performed on the measured data and it was observed that the maximum stress amplitude obtained by the simplified-EVE and NP methods were 87.8 and 82.8 MPa, respectively. The MCS technique was used in advance to identify the probability of occurrence for extreme stress amplitude and the results were in range. Furthermore, the proposed approaches were verified using the mathematical analysis and it was found to be well-correlated, which equalled to 86.5 MPa. In order to convert the stress spectra into fatigue damage, the standard S–N curve was used to eventually estimate the fatigue life. Based on the estimation, both simplified-EVE and NP methods estimated a higher fatigue damage compared to the superposition technique, which were 6.59E−6 and 5.81E−6, respectively. Given our current knowledge, however, the simplified-EVE method was more reasonable rather than the NP technique, because the NP method was relied on the type and size of the RFM and can be conservative. Seeing on the cycle counted of the simplified-EVE method that was closer to the superposition method, which were 2.37E5 and 2.43E5 cycles, respectively.

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Author contributions All authors performed equally to formulate of the paper.

Compliance with ethical standards

Conflict of interest All authors declare that they have no conflict of interest.

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