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Mathematical model for determining the viscoelastic properties of soft tissues using indentation tests

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1. Introduction

Indenting is a mechanical stress test where a very hard indentor is pressed into the material under investigation. It takes use of a hard tip whose geometrical and mechanical properties are known. In a measurement, load placed on the indenter tip is progressively increased, until it reaches a user defined value F_{max}. This load may be held constant for a period and is then gradually removed again. The periods of loading, holding and unloading the sample are user defined and recorded along with measured data. The course of the load curve for the viscoelastic material is shown in Fig. 1.

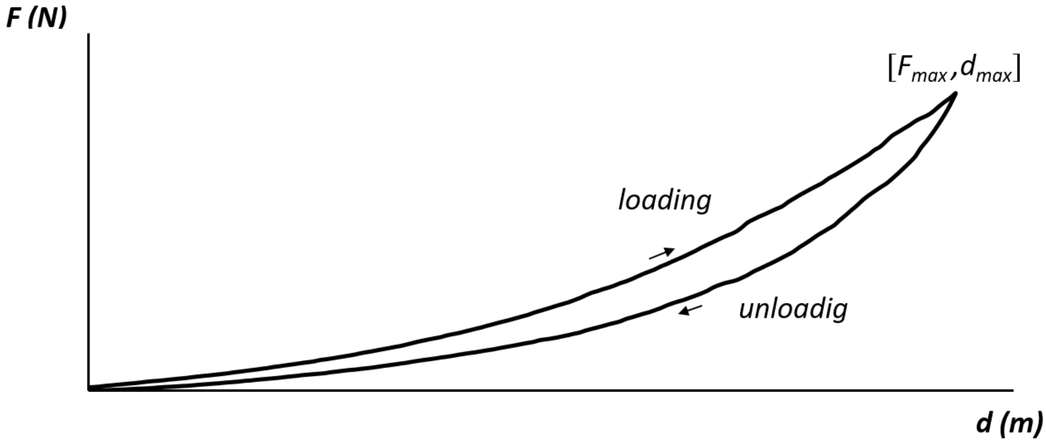

Fig. 1. A typical course of the load curve when indenting a viscoelastic material by a cylindrical indentor

The method is used especially for the testing of mechanical properties of materials at micro and nano scale. According to available sources, mathematical models are based on the theory of small deformations and developed especially to determine the hardness of the material. The question is whether it is possible to extend this reflection to a macroscopic scale and examine the viscoelasticity of materials? Such a concept could be used in medicine to objectify palpation examination of soft tissues, especially muscles. Instruments based on this principle already exist. They are called myotonometers.

2. Aim

The aim of the thesis was to create a mathematical model for the determination of viscoelastic properties of soft tissues using indentation stress tests.

3. Methods

Problem is formulated for indentation of a solid cylinder penetrating into an infinite half-space of viscoelastic material. The task is solved first, provided that the material is only elastic and with small deformations. The found shape of the deformation is used in the next step to formulate the tensor of deformation for the Neo-Hooke's hyperelastic material. The geometry of the task is shown in Fig. 2.

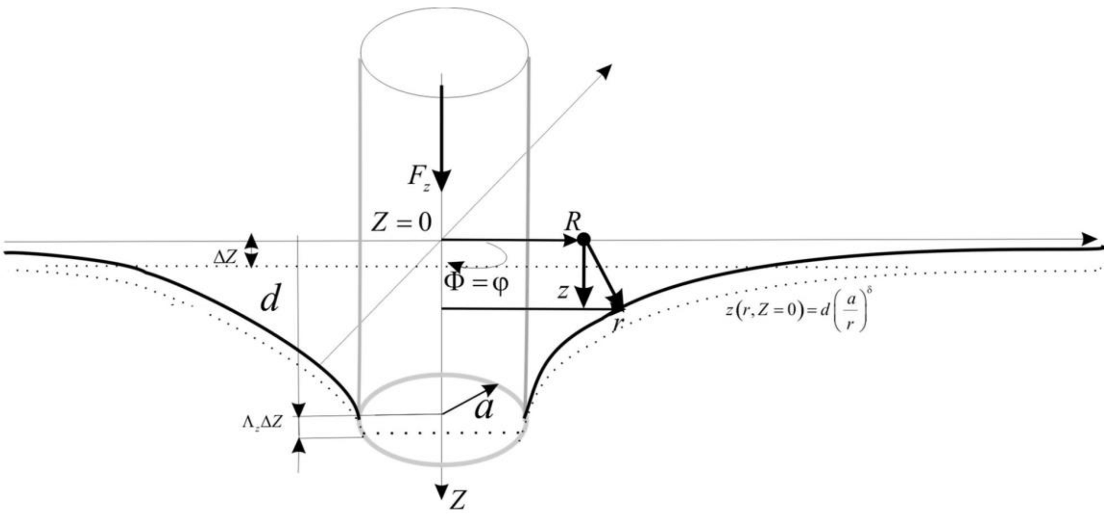

Fig. 2. Penetration of the solid cylinder into half-space by the action of point surface forces $\boldsymbol{F} = (0; 0; F_Z)$

4. Solution and results

During each deformation the weight is retained. Deformation gradient $F^i{}_I$ is used to determine deformation tensor. The change in density ρ is related to the change in volume (1-j). The volume change is proportional to the pressure p, K is the compressibility modulus (1)

$$\begin{aligned} &\rho_o = j\rho, \quad j = \det\left|F^i{}_I\right|, \quad F^i{}_I = \frac{\partial x^i}{\partial X^I}, \\ &p = K(1-j) = K\left(1 - \frac{\rho_o}{\rho}\right). \end{aligned} \tag{1}$$

The constitutive equation for the homogeneous elastic material (2a) can be generalized to the viscoelastic material (2b), [3]. In loading tests is typically measured elastic modulus E and Poisson's ratio σ

$$\begin{aligned} &a) \quad t^{ij} = K e_{(1)} \delta^{ij} + 2\hat{\mu}\overset{(o)}{e^{ij}} && K = \frac{E}{3(1-2\sigma)}, \quad \hat{\mu} = \frac{E}{2(1+\sigma)} \\ &b) \quad t^{ij} = K e_{(1)} \delta^{ij} + 2\hat{\mu}\overset{(o)}{e^{ij}} + 2\mu \overset{(o)}{d^{ij}} && K\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right], \ \hat{\mu}\left[\frac{\mathrm{J}}{\mathrm{m}^3}\right], \ \mu\ [\mathrm{Pa}\cdot\mathrm{s}] \end{aligned} \tag{2}$$

Another generalization is the hyperelastic material (Neo-Hook's). It is especially suitable for low compressible materials [4] and is also useful for describing biological tissues [1]. For the geometry of Fig. 2, it can be shown that the main components of Green's stress tensor are

$$t_{zz} = \frac{\hat{\mu}}{j^{5/3}}\left(\lambda^2 - \frac{j}{\lambda}\right), \qquad t_{xx} = t_{yy} = 0, \tag{3}$$

where λ is the elongation in the main direction of deformation. For small deformations ($d \sim 2a / 3$) can be used equation

$$\left(1-2\sigma\right)\Delta\mathbf{u} + \operatorname{grad}\operatorname{div}\mathbf{u} = 0, \tag{4}$$

where $\boldsymbol{u} = (u_x;\ u_y;\ u_z)$ is the displacement vector. Assuming the force under the cylindrical surface generates pressure

$$p\left(R\right) = p\left(0\right)\left(1-\frac{R^2}{a^2}\right)^{-1/2} \quad \text{for} \quad R \le a. \tag{5}$$

Analytical solution can be found using the Green function method [2]. It can be shown that the only non-zero displacement is in the z-direction (for $R \in \left(0, a\right)$) and is equal to

$$u_z = \frac{\pi\left(1-\sigma^2\right)a\,p\left(0\right)}{E} = d. \tag{6}$$

The indentation size is the same under the entire indentor area and corresponds to the depth of indentation d (Fig. 1 and Fig. 2).

$$F_z = \frac{2aE}{\left(1-\sigma^2\right)}d \tag{7}$$

Equation (7) is essential for determining the elastic material constants from the indentation test.

For large deformations, it is necessary to find Green's tensor of deformation $\boldsymbol{C}$ (10) and its own numbers (in cylindrical coordinates)

$$\begin{aligned} C_r &= \frac{a_{11}+\Lambda_z^2}{2} + \sqrt{\left(a_{11}-\Lambda_z^2\right)^2 + 4a_{31}^2}, \\ C_\varphi &= \left(\frac{r}{R}\right)^2, \\ C_z &= \frac{a_{11}+\Lambda_z^2}{2} - \sqrt{\left(a_{11}-\Lambda_z^2\right)^2 + 4a_{31}^2} \end{aligned} \quad \text{where} \quad \begin{aligned} a_{11} &= \left[1+\left(\delta\frac{d}{r}\left(\frac{a}{r}\right)^{\delta}\right)^2\right]\left(\frac{\partial r}{\partial R}\right)^2, \\ a_{31} &= -\left(\delta\Lambda_z\frac{d}{r}\left(\frac{a}{r}\right)^{\delta}\right)\left(\frac{\partial r}{\partial R}\right). \end{aligned} \tag{8}$$

Variable Λ_z represents compression of material in the z direction and δ is the surface curvature parameter. Next can be used (3).

To determine the viscosity of the material can use the (2b) and the Oldquist equation (4). For unidirectional load can be written

$$\begin{aligned} &t^{ij} = t^{ij}_{\text{el}} + t^{ij}_{\text{dis}}, \qquad \text{where} \qquad t^{ij}_{\text{el}} = Ke_{(1)}\delta^{ij} + 2\hat{\mu}\overset{(o)}{e}{}^{ij},\ \ t^{ij}_{\text{dis}} = 2\mu\overset{(o)}{d}{}^{ij}, \\ &\overset{(o)}{t}{}^{zz}_{\text{dis}} = 2\mu\overset{(o)}{d}{}^{zz} = \frac{4}{3}\mu_0\left(\frac{2}{3}\right)^n d_{zz}^{2n-1}, \qquad n \in \left\langle 0.5;1\right\rangle \end{aligned} \tag{9}$$

where μ_0 [Pa.s] is the coefficient of viscosity. The velocity deformation tensor d_{ij} can be obtained from the tensor of large deformations

$$2d_{ij} = \mathbf{F}^{-T}\dot{\mathbf{C}}\mathbf{F}^{-1}, \qquad \mathbf{C} = \mathbf{F}^T\mathbf{F} \tag{10}$$

for small deformations, then

$$d_{zz} = \dot{e}_{zz} = -\frac{8\dot{d}}{3\pi a} \approx \left(-\frac{8}{\pi}\frac{d}{a^2}\dot{d}\right) \tag{11}$$

5. Discussion

From the experimental point of view, it is difficult to determine the coefficients j, Λ_z, n and δ. The coefficient μ_0 can be determined from the velocity of indentation $\dot{d}$ at the corresponding stress.

6. Conclusion

The present study offers a theoretical analysis of indentation tests to determine the viscoelastic properties of soft tissues. The results can be used to objectify palpation examinations of the locomotor system by myotonometry.

The proposed concept can be further developed for FEM and layered composite materials. Can be used to study the mechanical properties of 3D structures (nonwovens, fibrous, nanofibrous and composite structures, or foam materials).

Acknowledgements

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