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Vibration analysis of a turbine blading with frictional inter-blade couplings

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Vibration analysis of interacting blades is an essential part in steam turbine design. However, modern large-scale finite-element-based computational models with a considerable number of degrees of freedom constitute challenges when used in dynamic simulations due to their complexity and time-consuming computations. Thus, various model reduction techniques are employed to lower the computational costs [1, 3]. In this contribution, a new generalized modal reduction method based on the complex modal values of the linearized nonconservative system is utilized.

Let us consider a rotating turbine blading with frictional inter-blade couplings (see Fig. 1). The corresponding equations of motion can be written in the form

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + (\boldsymbol{B} + \omega_0 \boldsymbol{G})\,\dot{\boldsymbol{q}} + \left[\boldsymbol{K}_S + \omega_0^2\,(\boldsymbol{K}_\omega - \boldsymbol{K}_{\mathrm{so}}) + \boldsymbol{K}_C\right]\boldsymbol{q} = \boldsymbol{f}_F\,(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{f}_E\,(t)\,, \qquad (1)$$

where $\boldsymbol{q}$ is the global vector of the generalized coordinates of all blades, $\boldsymbol{M}$, $\boldsymbol{B}$, $\omega_0\boldsymbol{G}$, $\boldsymbol{K}_S$, $\omega_0^2\boldsymbol{K}_\omega$ and $-\omega_0^2\boldsymbol{K}_{\mathrm{so}}$ are the block-diagonal matrices of mass, internal damping, gyroscopic effects, static stiffness, centrifugal stiffening and softening due to modelling in the rotating frame, respectively, $\boldsymbol{K}_C$ is the stiffness matrix corresponding to the linearized normal contact forces in couplings, $\boldsymbol{f}_F\,(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the vector of the nonlinear frictional forces and $\boldsymbol{f}_E\,(t)$ is the

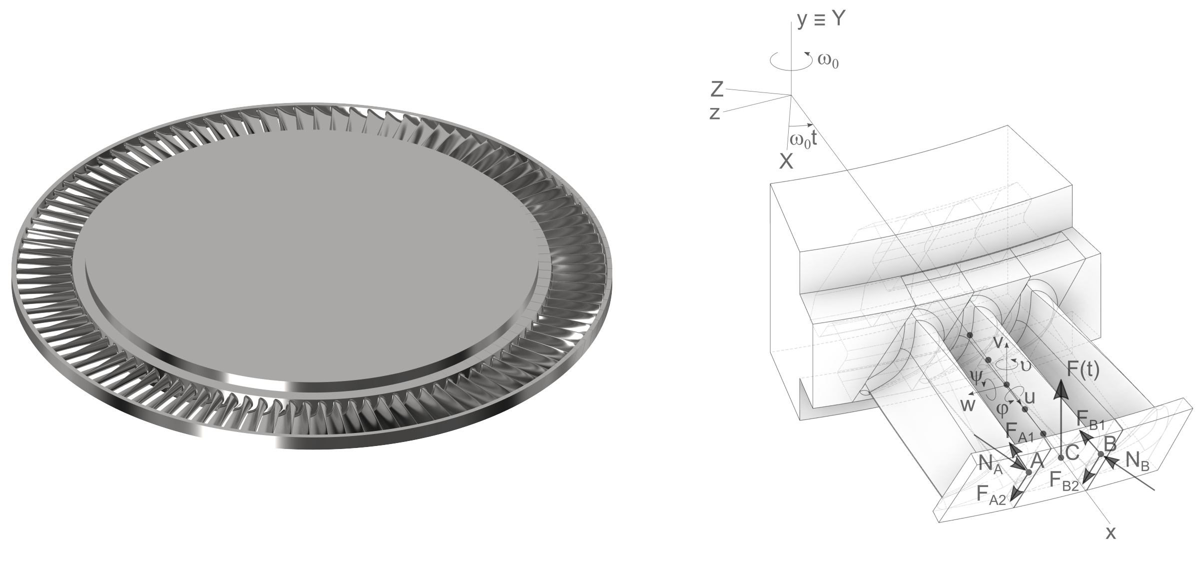

Fig. 1. (*Left*) Bladed turbine disk consisting of one hundred MTD30-HP15 blades; (*right*) drawing of a segment of the disk with contact forces

vector of the harmonic excitation forces. Eq. (1) may then be expanded to the state-space form

$$\boldsymbol{N}\dot{\boldsymbol{u}} + \boldsymbol{P}\boldsymbol{u} = \boldsymbol{p}, \tag{2}$$

where

$$\boldsymbol{u} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{q} \end{bmatrix}, \quad \boldsymbol{N} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{B} + \omega_0 \boldsymbol{G} \end{bmatrix},$$
$$\boldsymbol{P} = \begin{bmatrix} -\boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_S + \omega_0^2 \left(\boldsymbol{K}_\omega - \boldsymbol{K}_{\text{so}}\right) + \boldsymbol{K}_C \end{bmatrix}, \quad \boldsymbol{p} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{f}_F\left(\boldsymbol{q}, \dot{\boldsymbol{q}}\right) + \boldsymbol{f}_E\left(t\right) \end{bmatrix}. \tag{3}$$

In the first step, modal analysis of the linearized homogenous system

$$\boldsymbol{N}\dot{\boldsymbol{u}} + \boldsymbol{P}\boldsymbol{u} = \boldsymbol{0} \tag{4}$$

is performed. Complex right $\boldsymbol{U}$ and left $\boldsymbol{W}$ modal matrices (in the state space) can be written as

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{Q}\boldsymbol{\Lambda} \\ \boldsymbol{Q} \end{bmatrix}, \qquad \boldsymbol{W} = \begin{bmatrix} \boldsymbol{R}\boldsymbol{\Lambda} \\ \boldsymbol{R} \end{bmatrix}, \tag{5}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$, corresponding to complex spectral matrix $\boldsymbol{\Lambda}$, represent complex right and left modal matrices in generalized coordinates, respectively. Nevertheless, we consider only the reduced spectral matrix

$$\boldsymbol{\Lambda}_{\text{red}} = \text{diag}[\lambda_1, \dots, \lambda_R, \lambda_1^*, \dots, \lambda_R^*] = \text{diag}[\boldsymbol{\Lambda}_{\text{sub}}, \boldsymbol{\Lambda}_{\text{sub}}^*], \tag{6}$$

where λ_i, $i = 1, 2, \dots, R$, are selected (master) complex eigenvalues with positive imaginary parts and λ_i^* are their complex conjugates. Potential real eigenvalues are excluded. Corresponding right $\boldsymbol{U}_{\text{red}}$ and left $\boldsymbol{W}_{\text{red}}$ reduced modal matrices in the form

$$\boldsymbol{U}_{\text{red}} = \begin{bmatrix} \boldsymbol{Q}_{\text{red}}\boldsymbol{\Lambda}_{\text{red}} \\ \boldsymbol{Q}_{\text{red}} \end{bmatrix}, \qquad \boldsymbol{W}_{\text{red}} = \begin{bmatrix} \boldsymbol{R}_{\text{red}}\boldsymbol{\Lambda}_{\text{red}} \\ \boldsymbol{R}_{\text{red}} \end{bmatrix} \tag{7}$$

are composed of the matrices

$$\boldsymbol{Q}_{\text{red}} = \left[\boldsymbol{Q}_{\text{sub}}, \boldsymbol{Q}_{\text{sub}}^*\right], \qquad \boldsymbol{R}_{\text{red}} = \left[\boldsymbol{R}_{\text{sub}}, \boldsymbol{R}_{\text{sub}}^*\right], \tag{8}$$

where the submatrices $\boldsymbol{Q}_{\text{sub}}$ and $\boldsymbol{R}_{\text{sub}}$ contain master complex eigenvectors corresponding to the eigenvalues λ_i and the submatrices $\boldsymbol{Q}_{\text{sub}}^*$ and $\boldsymbol{R}_{\text{sub}}^*$ contain their corresponding complex conjugates.

In the second step, applying modal transformation

$$\boldsymbol{u} = \boldsymbol{U}_{\text{red}}\,\boldsymbol{x}, \tag{9}$$

where

$$\boldsymbol{x} = [x_1, \dots, x_R, x_1^*, \dots, x_R^*]^T = \begin{bmatrix} \boldsymbol{x}_{\text{sub}} \\ \boldsymbol{x}_{\text{sub}}^* \end{bmatrix} \tag{10}$$

is the vector of the master complex modal coordinates x_i, $i = 1, 2, \ldots, R$, and their complex conjugates x_i^*, and with regard to the biorthonormality conditions

$$\boldsymbol{W}_{\text{red}}^T \boldsymbol{N} \boldsymbol{U}_{\text{red}} = \boldsymbol{E}, \qquad \boldsymbol{W}_{\text{red}}^T \boldsymbol{P} \boldsymbol{U}_{\text{red}} = -\boldsymbol{\Lambda}_{\text{red}}, \tag{11}$$

where $\boldsymbol{E}$ is the identity matrix, Eq. (2) leads to

$$\begin{bmatrix} \dot{\boldsymbol{x}}_{\text{sub}} \\ \dot{\boldsymbol{x}}_{\text{sub}}^* \end{bmatrix} - \begin{bmatrix} \boldsymbol{\Lambda}_{\text{sub}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Lambda}_{\text{sub}}^* \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{\text{sub}} \\ \boldsymbol{x}_{\text{sub}}^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{\text{sub}}^T \\ \boldsymbol{R}_{\text{sub}}^H \end{bmatrix} \left[\boldsymbol{f}_F \left(\boldsymbol{q}, \dot{\boldsymbol{q}} \right) + \boldsymbol{f}_E \left(t \right) \right]. \tag{12}$$

Finally, the dynamic response in the generalized coordinates is, according to relations (3), (7), (8), (9) and (10), given by

$$\boldsymbol{q} = \boldsymbol{Q}_{\text{sub}} \boldsymbol{x}_{\text{sub}} + \boldsymbol{Q}_{\text{sub}}^* \boldsymbol{x}_{\text{sub}}^* = 2 \, \text{Re} \left(\boldsymbol{Q}_{\text{sub}} \boldsymbol{x}_{\text{sub}} \right), \tag{13}$$

$$\dot{\boldsymbol{q}} = \boldsymbol{Q}_{\text{sub}} \boldsymbol{\Lambda}_{\text{sub}} \boldsymbol{x}_{\text{sub}} + \boldsymbol{Q}_{\text{sub}}^* \boldsymbol{\Lambda}_{\text{sub}}^* \boldsymbol{x}_{\text{sub}}^* = 2 \, \text{Re} \left(\boldsymbol{Q}_{\text{sub}} \boldsymbol{\Lambda}_{\text{sub}} \boldsymbol{x}_{\text{sub}} \right). \tag{14}$$

The presented methodology was used to perform vibration analysis of the high-pressure turbine blading consisting of one hundred MTD30-HP15 blades [2] (see Fig. 1), where three different values of the coefficient of friction f were compared (see Fig. 2). It proved to be a valuable tool and provided a computationally cheap approach without incurring significant loss of accuracy.

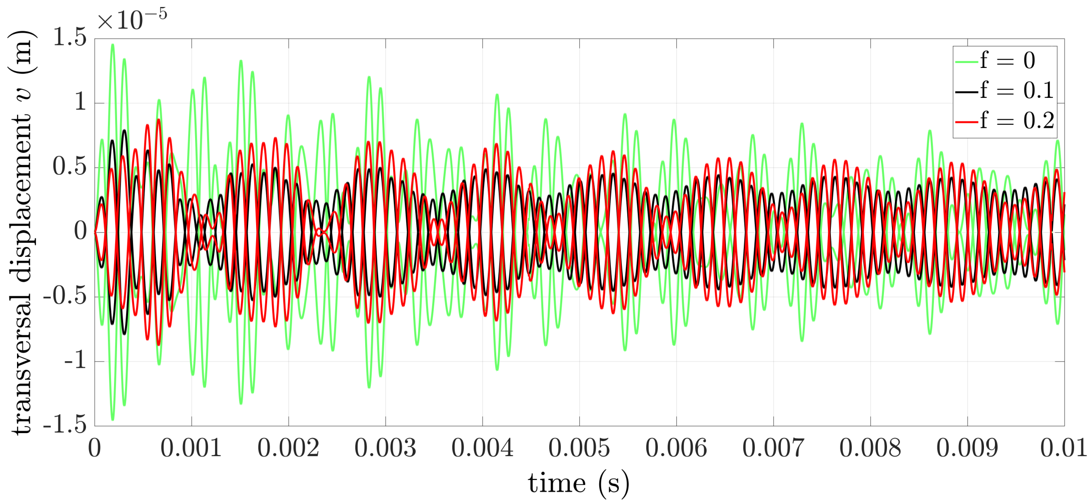

Fig. 2. Dynamic response of the blades to harmonic excitation for three different values of the coefficient of friction f

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