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Application of semi-analytical solution for transient wave propagation in 1D layered medium to various optimisation problems

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The main aim of this work is to solve the problem of transient waves propagated through a 1D layered elastic medium and to find an effective tool for solving optimisation problems for such type of heterogeneous medium. First, the semi-analytical solution for waves induced in a layered elastic thin rod by an axial impact is presented. Then, using the experimental data, three selected optimisation problems are solved by using an in-house Matlab code. Specifically, the inverse problems of material and geometrical properties and of loading pulse identification are solved followed by the problem of optimal design of layers to reduce the effect of impact.

The investigation of non-stationary response of a 1D layered medium made of n homogeneous elastic layers (see Fig. 1) can be handled by different methods. Probably the simplest way is to apply the existing solution for homogeneous thin rod (see e.g. [3]) to each ith layer ($i = 1, \ldots, n$) and to use the "stick" boundary conditions defined at each of $n - 1$ rods' interfaces. Using the Laplace transform and taking into account zero initial conditions of the problem, the wave equation for each ith homogeneous 1D rod leads to a simple ordinary differential equation. Introducing the Laplace transform of the axial displacement $U_i(x, p)$ and the transform of the axial stress $\Sigma_i(x, p)$, one can write the solution of such ith equation as

$$U_i(x_i, p) = A_i(p) \sinh\left(\frac{p x_i}{c_{0,i}}\right) + B_i(p) \cosh\left(\frac{p x_i}{c_{0,i}}\right), \tag{1}$$

$$\Sigma_i(x_i, p) = \frac{E_i p}{c_{0,i}} \left[A_i(p) \cosh\left(\frac{p x_i}{c_{0,i}}\right) + B_i(p) \sinh\left(\frac{p x_i}{c_{0,i}}\right)\right], \tag{2}$$

where x_i denotes the local coordinate (see Fig. 1), E_i is the Young's modulus, $c_{0,i}$ represents the wave speed, p is the complex variable and $A_i(p)$ and $B_i(p)$ are complex functions dependent on boundary conditions. These conditions can be formulated for the free (or fixed) end at $x_n = 0$ and for the excited end at $x_1 = l_1$ in a standard way (see e.g. [3]). The remaining $2(n - 1)$

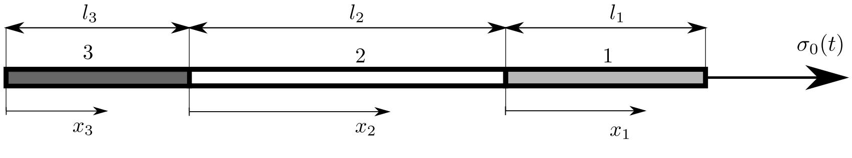

Fig. 1. Layered 1D media composed of three homogeneous mediums ($n = 3$)

conditions for the interfaces can be defined in Laplace domain as follows:

$$U_i(l_i, p) = U_{i-1}(0, p) \quad \text{and} \quad \Sigma_i(l_i, p) = \Sigma_{i-1}(0, p) \quad \text{for} \quad i = 2, \ldots, n. \tag{3}$$

Introducing the Laplace transforms (1) - (2) into the mentioned conditions, a system of $2n$ complex equations for the unknowns $A_i(p)$ and $B_i(p)$ is obtained. Basically, this system can be solved exactly only for very simple cases, so the numerical approach needs to be used for a general case. Once the values of $A_i(p)$ and $B_i(p)$ are determined, the problem of inverse Laplace transform has to be resolved. In this work, numerical inverse Laplace transform based on fast Fourier transform and Wynn's ε-algorithm (see [2]) are used. This approach delivers very precise results with low computational demands (see [1]), which makes it suitable for solving optimisation problems.

There are three optimisation problems solved in this work. Two of them are the inverse problems based on the acceleration response measured at free end of an excited rod made of three layers. The side layers were identical from the geometrical and material point of view while the central one was made of a different material and has different length. The identification of Young's modulus of all three layers is the first optimisation problem considered. This problem was solved using the standard Matlab function *fmincon* to very satisfying and accurate results. More than forty optimisations have been run for different lengths of the center layer and for different shapes of the loading pulse with a relative deviation of identified moduli under 5%.

The identification of the loading pulse is the second inverse problem solved. For this purpose, the pulse was approximated by Fourier sine series and its coefficients and base frequency were then the optimised parameters. Using more complex procedures based on the particle swarm optimisation (PSO) and on the genetic algorithm (GA), it was possible to obtain very accurate results already for 14 terms of the Fourier series. It is obvious from Fig. 2, where the identified pulse is compared to the measured one.

The last optimisation problem consists in the minimisation of stress amplitude at free end of the layered rod by designing the material parameters (material densities ρ_i and moduli E_i) and the lengths l_i of the layers. This problem was also solved using PSO and GA in Matlab

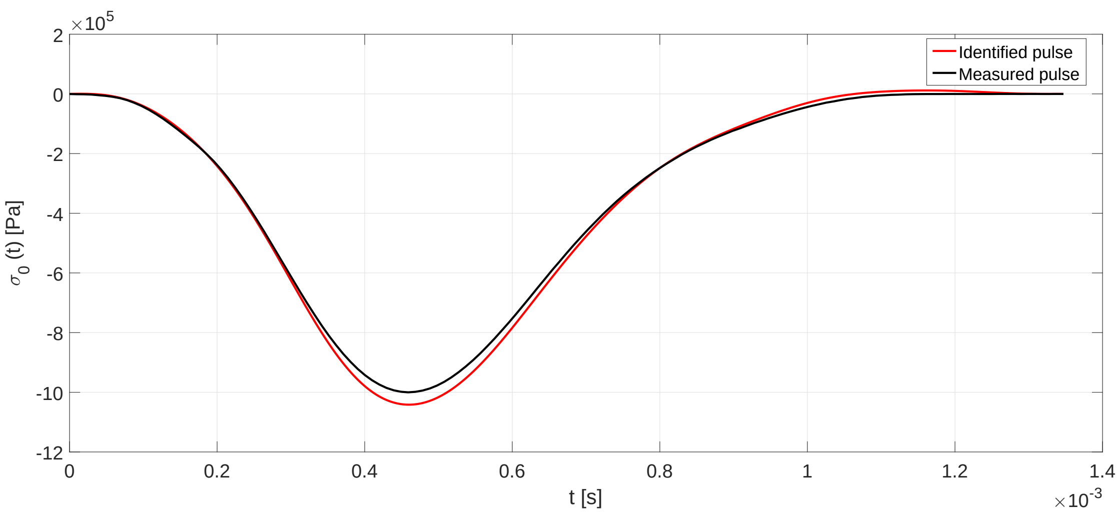

Fig. 2. Comparison of measured and identified pulse

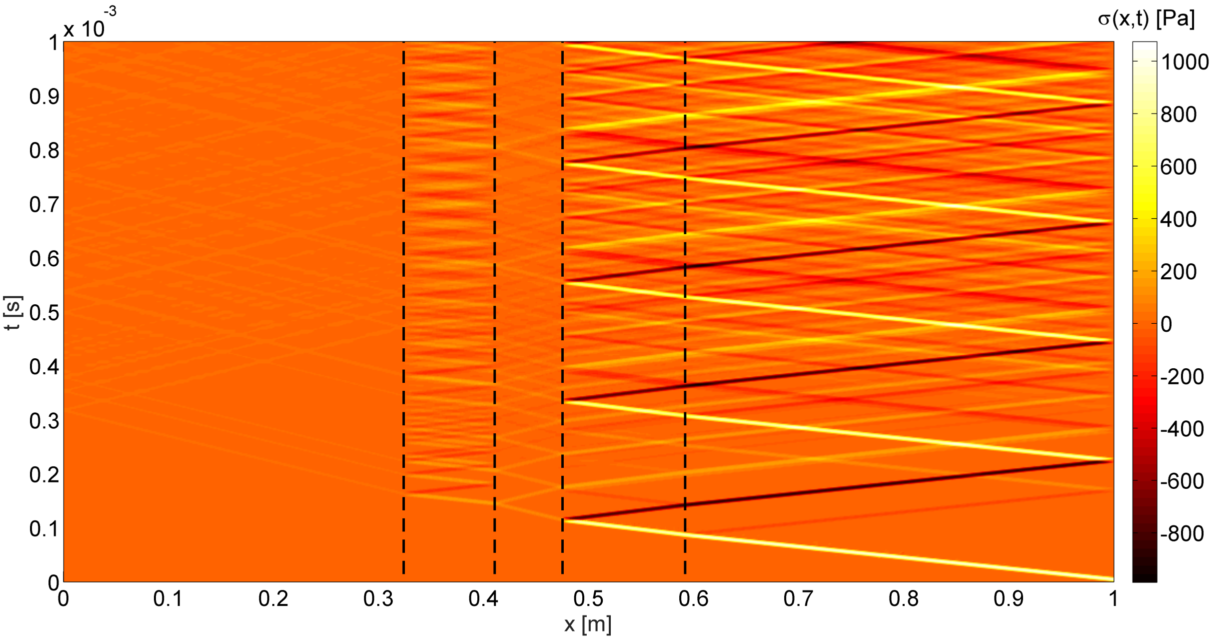

Fig. 3. Spatio-temporal distribution of axial stress $\sigma(x,t)$ in optimised rod composed of 5 layers

environment. Optimisations running over the same intervals of optimised parameters led to similar measure of initial pulse reduction but with different layer properties and composition. This problem, as formulated, has a lot of local minima and it was not possible to find one optimal solution by using mentioned procedures. The value of axial stress at free end of the rod was reduced to 1% - 30% of the initial pulse amplitude depending on the pulse duration. One example of a five-layered rod with optimised composition is presented in Fig. 3. This spatio-temporal distribution of axial stress $\sigma(x,t)$ in the rod clearly shows the primary tensile pulse propagating from the right excited end, interacting with the layer's boundaries (dashed lines) and reaching the free end of the rod with the amplitude about 3.6% of its initial value.

This work gives an overview of the fast solver developed for problems of non-stationary wave propagation in a 1D layered elastic media based on the semi-analytical approach. Using the experimental data, the solver is then effectively used for solving selected optimisation problems and the advantages and disadvantages of the proposed procedure are shown. Presented technique can also be used for the approximate solution of more general 1D heterogeneous problems, e.g. for the investigation of transient waves in functional graded materials.

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References

[1] Adámek, V., Valeš, F., Červ, J., Numerical Laplace inversion in problems of elastodynamics: Comparison of four algorithms, Advances in Engineering Software 113 (2017) 120-129.

[2] Brančík, L., Programs for fast numerical inversion of Laplace transforms in Matlab language environment, Proceedings of the 7th MATLAB Conference, Prague, 1999, pp. 27-39.

[3] Graff, K.F., Wave motion in elastic solids, Dover Publications, Inc., New York, 1991.