The Concept of Applying Class E Resonant Inverter to Induction Heating of Railway Turnouts

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Abstract:
The article discusses class E zero-voltage-switching resonant inverter (ZVS) to be applied in induction heating of railway turnouts. The resonant circuit of the inverter is subject to mathematical analysis using the method of state variables. Results of simulation testing, based on Simulink and Simplorer software, of an inverter at the operating frequency of 100 kHz are presented.

1. INTRODUCTION

Out of the many systems of snow removal and heating of turnouts, the stationary heating systems: electrical, gas, water circulation, and pneumatic turnout clearing systems are most often applied in European railways. The electrical heating on the basis of pipe, flattened-oval resistor heaters in metal sheaths is the predominant stationary system. Choice of a turnout clearing system is affected by the following factors: climate, efficiency of individual snow removal systems, reliability of system operation, cost of energy media, cost of maintaining the equipment, system's potential for automation. The impact of the foregoing factors varies depending on a rail administration and climate conditions.
The only heating system used in Poland is the electric heating with the aid of flattened-oval 330W/m radiators supplied with 230V AC voltage [1]. A significant drawback of electrical heating using AC network voltage is its low watt-hour efficiency.
The efficiency of turnout heating can be increased by applying of the induction heating method, where high-frequency magnetic field energy is converted into heating energy by means of rotary currents. High-frequency currents can be generated by resonant inverters supplied from a dc source [5]. Resonant inverters are characteristically capable of operating at high frequencies. Efficiency and reliability of the inverters depend largely on the course of switching of the power electronic components applied. In the processes of turn on and turn off, power losses occur as a result of multiplication of the current and the voltage in the semi-conductor components being switched. Consequently, attempts are made to implement switching processes at zero current or zero voltage [2, 3, 5, 11].
This article will discuss a class E zero-voltage-switching resonant inverter supplied from a voltage source. The inverter's mathematical circuit is described with the aid of state variables. The state variables are analysed using Simulink software. Results of simulation testing of the inverter with matching circuits based on Simplorer software are also provided.

2. TOPOLOGY AND OPERATION OF THE INVERTER

The circuit of the class E resonant inverter ZVS with one switch is shown in Figure 1.
High-inductance reactor L_d in series connection with the source of voltage E supplies current to the inverter. MOSFET transistor turns on and off at operating pulsation ω_T. Diode D is an integral part of the transistor and enables bi-directional current conductance through the transistor. The resistor R is an AC load. Inductance L, capacitance C, and resistance R form a series resonant circuit. Capacitance $C1$ includes: parasitic capacitances of the reactor L_d and of connections, as well as the output capacitance of the transistor.

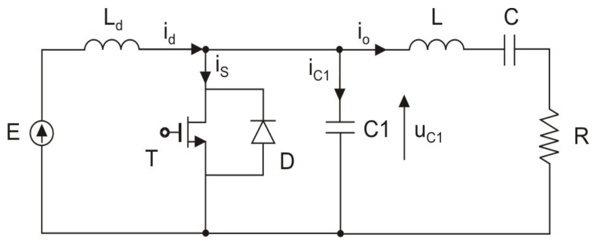

Fig. 1: Class E resonant inverter ZVS

When the switch is on, capacitance $C1$ is short-circuited by the transistor at on-resistance R_T (R_T << R). Series resonant circuit consists of inductance L, capacitance C, load resistance R, and transistor on-resistance R_T. Pulsation ω_{01} of R, R_T, L, C circuit is [8]:

$$\omega_{01} = \sqrt{\frac{1}{LC} - \left(\frac{R + R_T}{2L}\right)^2} \qquad (1)$$

and the loaded quality factor Q_{01} of R, R_T, L, C circuit equals:

$$Q_{01} = \frac{\omega_{01} L}{(R + R_T)} = \frac{1}{\omega_{01} C (R + R_T)} \qquad (2)$$

When the switch is off, the series resonant circuit consists of inductance L, capacitances C and $C1$, and load resistance R. The capacitances C and $C1$ are in a series connection, and the equivalent capacitance C_Z is lower than C and $C1$:

$$C_Z = \frac{C \cdot C1}{(C + C1)} \tag{3}$$

Pulsation ω_{02} of $R, L, C, C1$ circuit is:

$$\omega_{02} = \sqrt{\frac{1}{L \cdot C \cdot C1/(C + C1)} - \left(\frac{R}{2L}\right)^2} \tag{4}$$

and the loaded quality factor Q_{02} of $R, L, C, C1$ circuit equals:

$$Q_{02} = \frac{\omega_{02} \cdot L}{R} = \frac{1}{\omega_{02} \cdot R \cdot C \cdot C1/(C + C1)} \tag{5}$$

The inverter can operate in three cases of operation which depend on the value of load resistance R:

- at $R = R_{opt}$ the inverter operates in the range of optimum work. The transistor is turned on and turned off at zero voltage, and turned on at zero current (soft commutation) [5].
- at $R < R_{opt}$ the inverter operates in the range of sub-optimum work. The transistor is switched at zero voltage (soft commutation) and at hard current commutation. Power losses during transistor switching are higher than in the case of optimum work, and the output power of the inverter is lower. The amplitude of load current is lower than in the case of optimum work.
- at $R > R_{opt}$ the inverter operates in the range of non-optimum work. The transistor is turned on at hard voltage and current commutation, and turned off at zero voltage (soft commutation) and at hard current commutation. Power losses during transistor switching are higher than in the case of optimum and sub-optimum work.

On the basis of [2], values of R, L, $C1$, C, L_d of the circuit illustrated in Figure 1 are:

$$R = \frac{8P_0}{(\pi^2 + 4)I_d^{\,2}} \tag{6}$$

$$L = \frac{Q_0 \cdot R}{\omega_T} \tag{7}$$

$$C1 = \frac{I_d^{\,2} \cdot \eta}{\pi \cdot P_0 \cdot \omega_T} \tag{8}$$

$$C = \frac{1}{\eta\left(Q_0 - \frac{\pi(\pi^2 - 4)}{16}\right)R} \tag{9}$$

$$L_d = \frac{E \cdot T}{2 \cdot \Delta I_d} \tag{10}$$

where:
P_0 – output power of the inverter,
I_d – average value of the supply current of the inverter,
Q_0 – the quality factor of the resonant circuit,
ω_T – operating pulsation of the transistor,
η – ratio of the inverter efficiency,
T – the inverter operation period,
ΔI_d – acceptable current change I_d during the period T.

There is only one set of parameters: L, C, $C1$, L_d, R, where optimum work is effected for a given transistor on switch duty cycle d.

For purposes of the analysis, the following assumptions are made:

- The inverter operates in the range of optimum work, i.e. $R=R_{opt}$.
- The transistor on switch duty cycle d equals 0.5 [5].
- Transistor and the backward diode form a switch whose on-resistance equals R_T, off-resistance is infinity. Switching times of the switch are zero.

Analysis of the inverter's resonant circuit will be stated in relative units; base quantities of the voltage, current, and time are respectively: E, $E\omega_T C$, $1/\omega_T$.

$$\begin{cases} u^* = \dfrac{u(t)}{E} \\ i^* = \dfrac{i(t)}{E \cdot \omega_T \cdot C} \\ \tau = \omega_T \cdot t \end{cases} \tag{11}$$

where: u^* – voltage in relative units,
i^* – current in relative units,

Cycle of the inverter's operation is divided into two intervals (Fig.4). During the first interval: ($0 \le \tau \le \pi$), the transistor is on.

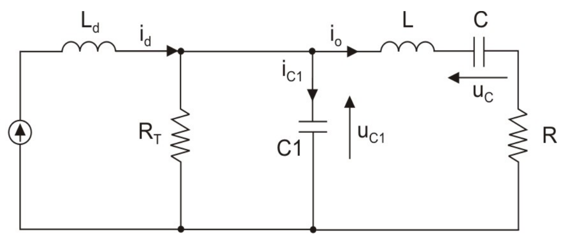

Fig. 2: Resonant circuit for the first interval of the inverter's operation

Resonant circuit for the first interval (Fig.2), is given by differential equations:

$$\begin{cases} u_{C1} = i_0 R + L\dfrac{di_0}{dt} + u_C \\ i_d = \dfrac{u_{C1}}{R_T} + i_0 + C1\dfrac{du_{C1}}{dt} \\ i_0 = C\dfrac{du_C}{dt} \\ E - L_d\dfrac{di_d}{dt} - u_{C1} = 0 \end{cases} \tag{12}$$

The system of equations (12) in relative units is:

$$\begin{cases} \dfrac{di_0^*}{d\tau} = \dfrac{1}{P_1Q_1}u_{C1}^* - \dfrac{1}{Q_1}i_0^* - \dfrac{1}{P_1Q_1}u_C^* \\ \dfrac{di_d^*}{d\tau} = \dfrac{1}{YP_1Q_1} - \dfrac{1}{YP_1Q_1}u_{C1}^* \\ \dfrac{du_{C1}^*}{d\tau} = \dfrac{1}{D}i_d^* - \dfrac{1}{P_2}u_{C1}^* - \dfrac{1}{D}i_0^* \\ \dfrac{du_C^*}{d\tau} = i_0^* \end{cases} \tag{13}$$

where:

$$Q_1 = \frac{\omega_T L}{R},\; P_1Q_1 = \omega_T^2 LC,$$
$$P_2 = \omega_T R_T C1,\; D = \frac{C1}{C},\; Y = \frac{L_d}{L} \tag{14}$$

Introducing designations of state variables:

$$\begin{cases} i_0^*(\tau) = x_1(\tau) \\ i_d^*(\tau) = x_2(\tau) \\ u_{C1}^*(\tau) = x_3(\tau) \\ u_C^*(\tau) = x_4(\tau) \end{cases} \tag{15}$$

and marking:

$$a = \frac{1}{Q_1},\; b = \frac{1}{P_1Q_1},\; c = \frac{1}{YP_1Q_1},$$
$$d = \frac{1}{D},\; e = \frac{1}{P_2} \tag{16}$$

equations of state are:

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_3 - bx_4 \\ \dot{x}_2 = -cx_3 + c \\ \dot{x}_3 = -dx_1 + dx_2 - ex_3 \\ \dot{x}_4 = x_1 \end{cases} \tag{17}$$

It is assumed that the initial conditions for the first interval of the first cycle of the inverter's operation are zero. The first operating interval finishes when $\tau=\pi$, and $u_{C1}^* = 0$.

During the second interval of the inverter's operation: $(\pi < \tau \le 2\pi)$ the transistor is off.

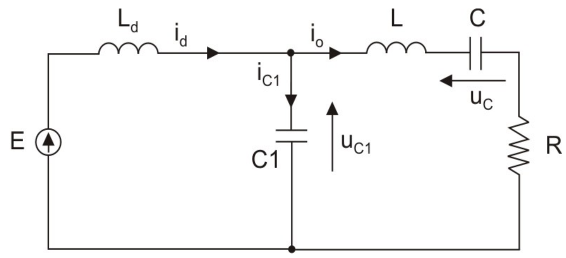

Fig. 3: Resonant circuit for the second interval of the inverter's operation

Resonant circuit for the second interval, (Fig.3), is described by differential equations:

$$\begin{cases} L\dfrac{di_0}{dt} + u_C + i_0 R - u_{C1} = 0 \\ i_d = C1\dfrac{du_{C1}}{dt} + i_0 \\ i_0 = C\dfrac{du_C}{dt} \\ E - L_d\dfrac{di_d}{dt} - u_{C1} = 0 \end{cases} \tag{18}$$

The system of equations (18) in relative units is:

$$\begin{cases} \dfrac{di_0^*}{d\tau} = \dfrac{1}{P_1Q_1}u_{C1}^* - \dfrac{1}{P_1Q_1}u_C^* - \dfrac{1}{Q_1}i_0^* \\ \dfrac{di_d^*}{d\tau} = \dfrac{1}{P_1Q_1Y} - \dfrac{1}{P_1Q_1Y}u_{C1}^* \\ \dfrac{du_{C1}^*}{d\tau} = \dfrac{1}{D}i_d^* - \dfrac{1}{D}i_0^* \\ \dfrac{du_C^*}{d\tau} = i_0^* \end{cases} \tag{19}$$

Introducing denotations of state variables (15), equations of state for the second interval are:

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_3 - bx_4 \\ \dot{x}_2 = -cx_3 + c \\ \dot{x}_3 = -dx_1 + dx_2 \\ \dot{x}_4 = x_1 \end{cases} \tag{20}$$

Waveforms in subsequent cycles of the inverter's operation are determined in a manner analogous to the method in the first cycle [7]. The final conditions of the successive operation cycle become the initial conditions for the following cycle of the inverter's operation. Current and voltage waveforms in relative units are shown in Figure 4.

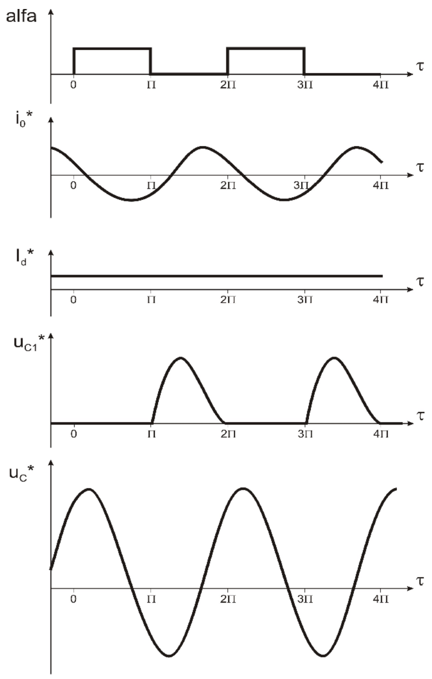

Fig. 4: Current and voltage waveforms of the inverter during optimum work obtained on the basis of Simulink software (relative units)

3. COOPERATION OF THE INVERTER AND A MATCHING CIRCUIT

A matching circuit is a four-terminal network containing passive elements, incorporated between the load and the inverter. Application of a matching circuit is intended to take maximum advantage of the transistor and to reduce energy losses at variable load of the inverter. In most industrial solutions, load resistance R assumes values which are variable and different than the optimum value for a given resonant circuit. At the load resistance $R \neq R_{opt,}$ the switching losses in the inverter are the greater, the higher the operating frequency of the circuit. Figure 5 shows a ZVS inverter with a matching circuit that enables transistor switching at zero voltage for the load resistance $R \neq R_{opt}$

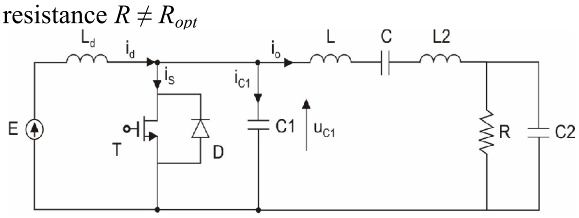

Fig. 5: Resonant inverter ZVS with a matching circuit

The matching circuit in Figure 5 consists of inductance $L2$ and capacitance $C2$. Values of the elements $L2$, $C2$ should be selected in such a way that the following dependency can obtain:

$$R_{op} = \frac{R_o \cdot \dfrac{1}{j\omega_T \cdot C2}}{R_o + \dfrac{1}{j\omega_T \cdot C2}} + j\omega_T \cdot L2 \qquad (21)$$

Values of the matching circuit are related by:

$$L2 = \frac{R_{op}}{\omega_T}\sqrt{\frac{R}{R_{op}} - 1} \qquad (22)$$

$$C2 = \frac{\sqrt{\dfrac{R}{R_{op}} - 1}}{\omega_T \cdot R} \qquad (23)$$

From equations (22), (23):

$$R = \frac{L2}{C2 \cdot R_{op}} \qquad (24)$$

The matching circuit permits the inverter to operate within the range of minimum switching losses at variable load resistance R.

4. SIMULATION RESULTS

The basic version of the resonant voltage-switched inverter (Fig.1) was subject to simulation testing based on Simulink and Simplorer software [6, 12]. The following data were assumed for purposes of designing the circuit: load power P_0=1000W, supply current I_d=10A, Q_0=7, operating frequency of the inverter f_T=100 kHz, circuit efficiency η=0.93, the transistor on switch duty cycle d=0.5. According to [1], the circuit of the resonant inverter contains elements of the following values: L=64.3μH, C=50.7nF, $C1$=47.1nF, L_d=3.8mH, E=141.3V, R=var. Simulink-based simulation model of the inverter is presented in Figure 6. Waveforms of: load current i_0^*, supply current i_d^*, voltage across the transistor u_{C1}^*, and voltage u_C^* across the capacitance C, in relative units for the range of optimum work, are observed in oscilloscopes i_0, i_d, u_{C1}, u_C, as a result of sequential switching of calculations between Subsystem blocks described with the equations of state (17) and (20).

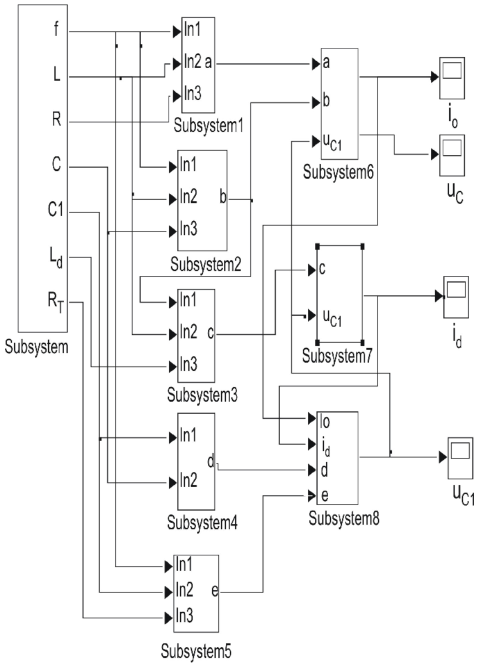

Fig. 6: Simulation model of the resonant inverter in Simulink software

Simplorer-based simulation model of the inverter is presented in Figure 7. MOSFET IRFM460 transistor was used in the simulation model [12]. To calculate capacitance $C1$, the value of transistor's output capacitance was considered C_{oss}=1000pF [9, 10].

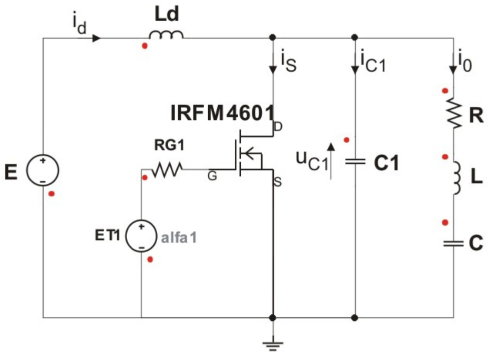

Fig. 7: Simulation model of the inverter in Simplorer software

Figure 8 presents simulation models of matching circuits that contain the following elements: $L2$, $C2$ for the inverter loaded with the resistance R=10Ω.

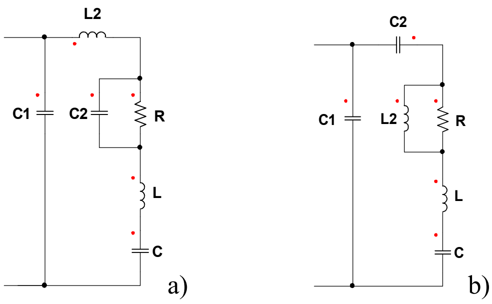

Fig. 8: Fig.8. Matching circuits for R=10Ω, f=100kHz, a) $L2$=7.5μH, $C2$=111nF, b) $L2$=22.7μH, $C2$=338nF

Figure 9 illustrates waveforms of currents and voltages which occur in the inverter cooperating with matching circuits (Fig.8). The resulting waveforms are identical after application of both the circuits. Due to lower values of $C2$, $L2$, a matching circuit whose configuration is shown in Figure 8a is more advantageous.

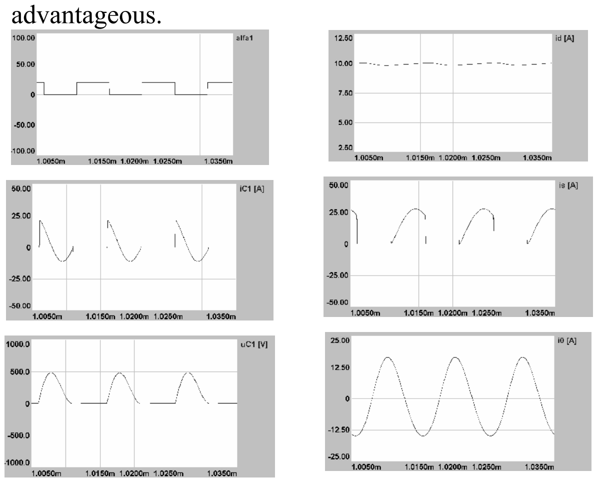

Fig. 9: Current and voltage waveforms resulting from simulation of the inverter with matching circuits (Fig.12), R=10Ω

5. CONCLUSION

Simulation tests of the inverter under analysis lead to the following conclusions:

- Maximum values of voltages across the transistor are about four times higher than the supply voltage, and restrain the scope of inverter's applications to operation at supply voltages whose values are limited by the value of transistor's drain – source voltage.
- Within the range of optimum operation, in the case of basic inverter topology, the transistor is switched at zero voltage and turned on at zero current, and power losses due to the switching are virtually non-existent.
- The range of the inverter's optimum operation relates to one value of load resistance at a given operating pulsation, which significantly limits potential for applications of basic topology inverters.
- The inverter with a matching circuit guarantees current operation with zero-voltage switching technique at variable values of the load resistance, and enables reduction of energy losses at switching.
- The inverter with a matching circuit generates sinusoid current in a broad range of frequencies, and can be applied in areas where heat processes should be characterised by high efficiency in a wide range of control, in particular, in induction heating of railway turnouts.
- It is estimated that application of the technique of induction heating using

resonant inverters to heating of rail turnouts will considerably enhance heating efficiency compared to electric heating powered with ac voltage of mains frequency. Implementation of group induction heating for a specific number of turnouts will permit to reduce the required power.

- Power circuits using induction heating, owing to their higher heating efficiency, will comprise smaller and lighter topologies of the heating systems when compared to resistance heating.

6. REFERENCES

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