COMPUTATIONAL MECHANICS 2021
36th conference with international participation
Srní
November 8 - 10, 2021

Influence of various constraints in topology optimization of tensegrity structures using mixed integer linear programming

R. Bulín[a], M. Hrabačka[b], M. Hajžman[a,b]

[a] NTIS – New Technologies for the Information Society, Faculty of Applied Sciences, Technická 8, 301 00 Plzeň, Czech Republic
[b] Department of Mechanics, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 301 00 Plzeň, Czech Republic

The definition of a tensegrity structure is not unique, but the word tensegrity itself is composed from words ”tension” and ”integrity”. This implies, that some continuous network of tension is needed to maintain the structure’s integrity [4]. In general, the tensegrity is a pre-stressed structural system that has a great potential to reach favorable strength to weight ratio. There are various types of tensegrities and they can be found throughout several different fields, such as architecture (e.g., lightweight self-supporting roofs, bridges or towers), biology (e.g., tensegrity cell model, skeleton with muscles) or engineering (lightweight manipulators, human exoskeletons, space deployable structures). From the perspective of mechanical engineering and robotics, the tensegrity structures are interesting systems in terms of their mechanical properties, such as stability, modal properties or controllability and dexterity. The first step before an utilization of tensegrity structure as a part of a mechanical system is to find suitable topology and form of a tensegrity. In this paper, the tensegrity topology optimization using mixed integer linear programming is briefly summarized and the influence of various optimization constraints is further discussed.

The topology optimization of tensegrity structures is based on the ground structure method [1] which is commonly used for topology optimization of truss pin-jointed structures. This method is based on the definition of nodes positions and on the definition of candidate members, each member connects two nodes. In case of tensegrity structures, each candidate member can be a strut, cable or it can be completely omitted. To distinguish the type of member, two binary optimization variables x_i, y_i for each member are used [2]. Since these variables can have only integer values 0 or 1 and commonly also other continuous variables are used, the whole optimization problem is formulated as a mixed integer linear programming (MILP) problem [1]. The linear programming is mainly used because of computational efficiency of numerical methods, that can operate with binary variables. The whole problem can utilize a large number of optimization variables (even for simple structures it can be more than thousand) and a large number of optimization constraints (again, more than thousand constraints for simple structures), so any nonlinearity can lead to difficulties with convergence in acceptable computational time.

Since the MILP formulation is used, the objective function is linear. The common optimization objectives are: (a) a minimization of number of cables used in the tensegrity, (b) a minimization of the total length of all cables [2], (c) a minimization of the overall structure weight [5], (d) a maximization of the sum of pre-stress forces in the structures [4]. First two objectives are similar and practical, because with the lower number of cables (or total cable length), the whole structure can be easier to manufacture. Since the pre-stress forces can be negative (compressive force) or positive, the fourth objective tends to generate a stiff and stable tensegrity structure.

The basic MILP formulation for the topology optimization of tensegrity structures is [2]

$$\min_{\mathbf{x},\mathbf{y},\mathbf{q}} \sum_{i \in E} y_i, \tag{1}$$

$$\text{s.t.} \quad \mathbf{H}\mathbf{q} = \mathbf{0}, \tag{2}$$

$$-q_s^U x_i \le q_i \le -q_s^L x_i + q_c^U (1 - x_i), \quad \forall i \in E, \tag{3}$$

$$q_c^L y_i - q_s^U (1 - y_i) \le q_i \le q_c^U y_i \quad \forall i \in E, \tag{4}$$

$$\sum_{i \in E(j)} x_i \le n \quad \forall j \in V. \tag{5}$$

Eq. (1) represents the objective function, which evaluates a number of cables in the whole structure. The optimization variables are binary variables $\mathbf{x}$ and $\mathbf{y}$ and they determine the type of member: if $(x_i,\ y_i) = (1,\ 0)$, the member i is strut, if $(x_i,\ y_i) = (0,\ 1)$, the member i is cable, if $(x_i,\ y_i) = (0,\ 0)$, the member is not used in the structure. Optimization variable $\mathbf{q}$ represents the pre-stress forces in members. Eq. (2) is the static equilibrium condition, where $\mathbf{H}$ is the equilibrium matrix [2]. Eqs. (3) and (4) ensure, that if member i is a cable, it will be in tension, and if member i is a strut, it will be in compression. Constants q_s^U, q_s^L, q_c^U and q_c^L represent upper (U) and lower (L) boundaries on forces in strut members (s) and cable members (c). Eq. (5) expresses so called discontinuity conditions and it ensures, that in one structure node there will end n or less struts. For classical tensegrities, there is only one end of strut in one node, thus $n = 1$. Symbol E denotes a set of candidate members, V is set of nodes and $E(j)$ is a set of members, that ends in node j.

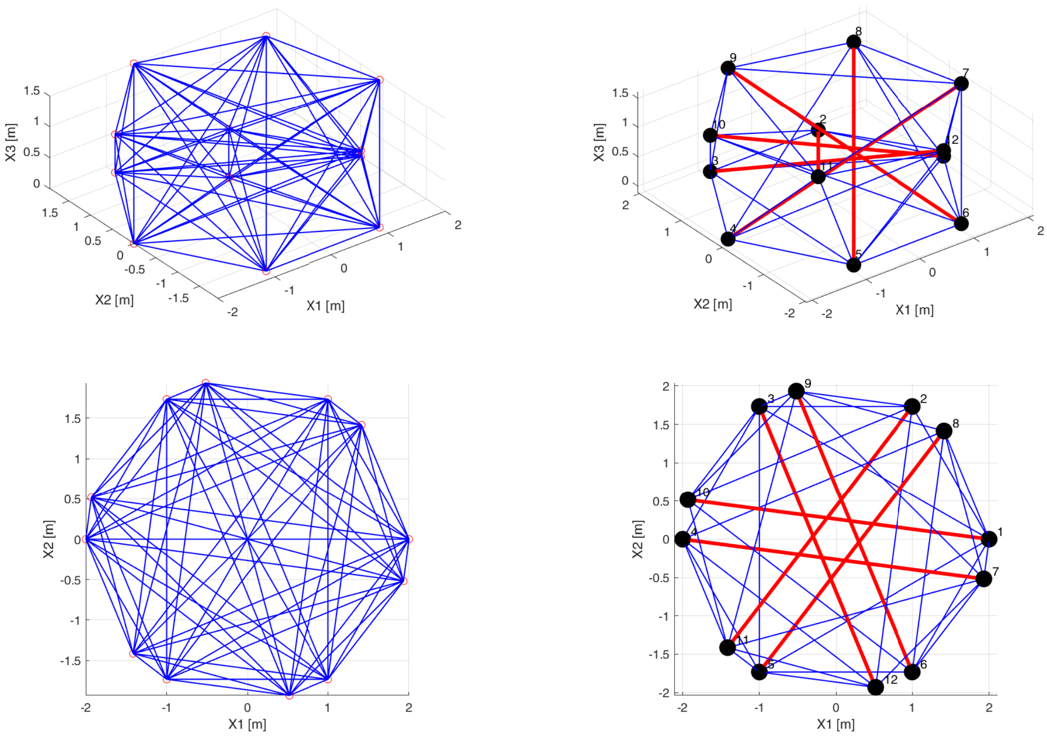

Fig. 1. *Left:* Ground structure of two hexagons (blue - candidate member); *Right:* Resulting tensegrity structure using struts with the same lengths (red - strut, blue - cable)

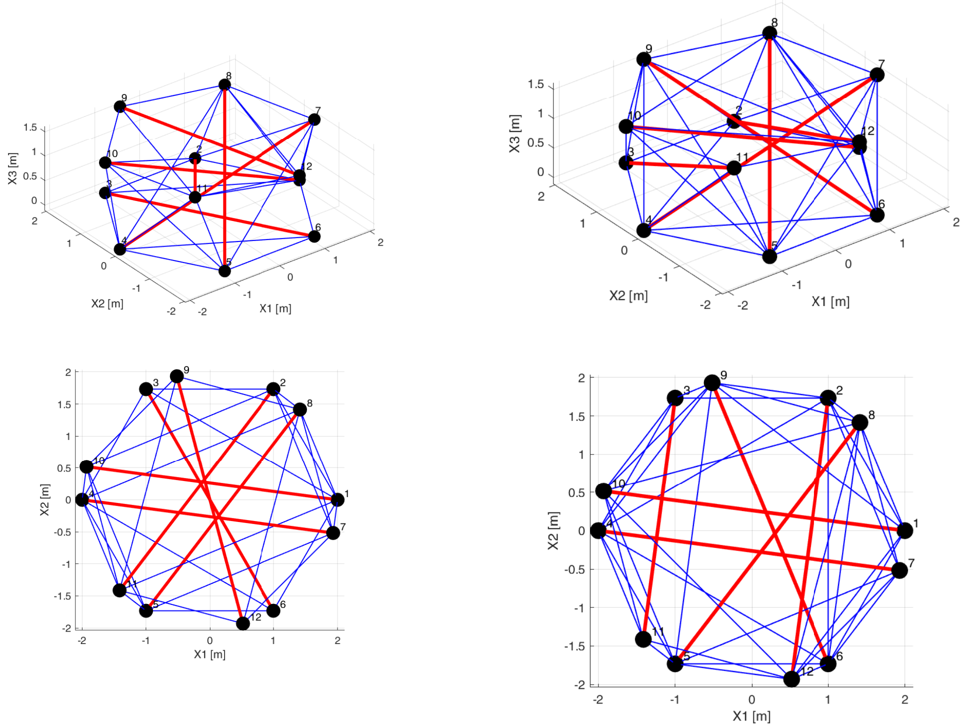

Fig. 2. *Left:* Resulting tensegrity structure using two different strut lengths (red - strut, blue - cable); *Right:* Resulting tensegrity structure using three different strut lengths (red - strut, blue - cable)

The optimization process does not often lead to satisfying results using Eqs. (1)–(5) only, thus many other additional constraints, that can also require to add other optimization variables, can be employed to obtain desired results. The additional constraints can be generally divided into two groups: practical constraints and properties constraints. Essential practical constraints are: constraints to avoid intersecting members [3], constraints on the number of strut length groups used [2] (this requires to sort the potential members by length and to create groups with similar length), constraints on kinematic and static indeterminacy [2] or constraints on the number of cables in one node. The tensegrity properties constraints are: constraints on the structural deformation when externally loaded [3] or constraints on the member forces to prevent buckling [5].

The influence of the number of strut length groups on the structure of hexagonal tensegrity is shown in Figs. 1 and 2. On the left side of Fig. 1 the ground structure with all candidate members is shown. The right side of Fig. 1 shows the structure after the topology optimization, where all the used struts belong to the same length group, thus they have the same length. The structure has a nice level of symmetry. In Fig. 2 the resulting structure with two resp. three strut length groups are shown.

In Fig. 3, the topology optimization of tensegrity tower is illustrated. At first, the counter-clockwise twisted triangular tensegrity is shown in order demonstrate one layer of the tensegrity tower. Then, the ground structure that is composed from two layers and the resultant topology after optimization are shown. The bottom layer of the tower is the counter-clockwise twisted triangular tensegrity and the upper layer is clockwise twisted triangular tensegrity. These results demonstrate, that the topology optimization reached the expected results. In addition, the constraints on the structure deformation were used in the optimization. It allows to specify the load in the nodes (in this case, nodes 10, 11 and 12 were loaded). In the results of the optimization, the forces in members that equalize the external loads are also obtained, which is an advantage in comparison with classical form-finding methods. It can be concluded, that the topology optimization of tensegrity structures using MILP formulation provides also practical results, such as total forces in the used members. By a combination of various constraints in topology optimization of tensegrity structures, different results can be obtained.

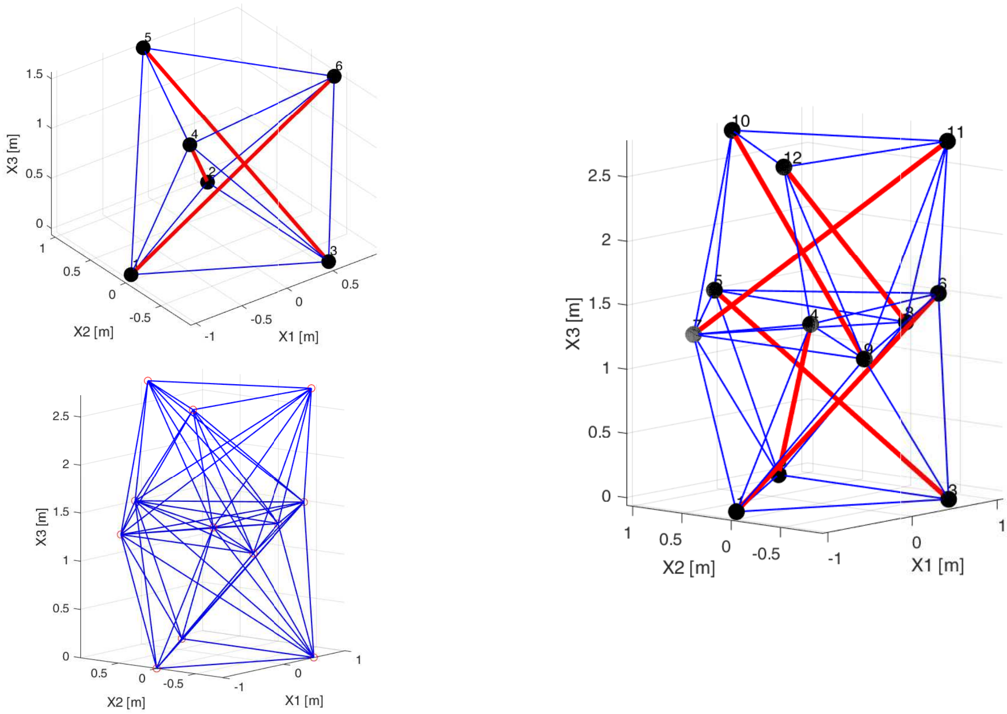

Fig. 3. Counter-clockwise triangular tensegrity, ground structure for tensegrity tower and resultant topology of tensegrity tower

Acknowledgement

The work was supported by the Czech Science Foundation project number 20-21893S.

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