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Implementation of a modified unified viscoplastic constitutive model

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A modified unified viscoplastic constitutive material model is proposed for simulating the temperature and time-dependent cyclic behaviour of the material SiMo 4.06. The constitutive model is based on the non-linear kinematic hardening rule of Chaboche, in which several features are added to incorporate strain rate sensitivity, the strain range dependency of the cyclic hardening, static recovery and mean stress evolution. In addition, continuum damage mechanics is incorporated into the constitutive model. This advanced constitutive model is numerically integrated and implemented into a finite element method analysis as well as into the stand-alone code to simulate the strain-controlled uniaxial low-cycle fatigue tests and the uniaxial creep tests.

1. Constitutive model

The constitutive model is based on previous version of this model [2]. The non-linear kinematic hardening rule of Chaboche with static recovery term and mean stress evolution can be expressed as follows:

$$\dot{\boldsymbol{\alpha}}_i = \frac{2}{3} C_i \dot{\boldsymbol{\varepsilon}}^{\boldsymbol{pl}}(1-D) - \gamma_i(\boldsymbol{\alpha}_i - \boldsymbol{Y}_i)\dot{p}(1-D) - \gamma_{ri}[J(\boldsymbol{\alpha}_i)]^{m_i-1}\boldsymbol{\alpha}_i + \frac{1}{C_i}\frac{\partial C_i}{\partial T}\boldsymbol{\alpha}_i\dot{T}\,. \tag{1}$$

To capture viscoplastic behaviour, a hyperbolic sine flow rule is chosen [1]

$$\dot{p} = \alpha \sinh\langle \beta f \rangle\,. \tag{2}$$

Strain range dependency of the cyclic hardening is achieved by introducing dependency of isotropic hardening parameter on the plastic strain memory surface radius. Then, continuum damage mechanics is incorporated into the constitutive model in order to simulate the tertiary creep strain responses and to predict fatigue damage evolution under uniaxial loading. Isotropic damage variable is evolved according to [3] as

$$\dot{D} = A\,\frac{(1-e^{-q})\sigma_{eq}^{p}}{q}\,e^{qD}\,. \tag{3}$$

The scalar isotropic damage model is coupled with the constitutive model through the effective stress relationship. This modified constitutive model has been numerically integrated and implemented into the finite element method analysis and experimentally validated against a broad set of fatigue, creep and fatigue-creep responses under isothermal temperature conditions. In the next section the development of a constitutive model is described for a stand-alone viscoplasticity code.

2. Stress return mapping

The numerical scheme used to integrate the differential equations is an implicit backward Euler method called the radial return method. The numerical scheme leads to a nonlinear scalar algebraic system of equations having scalar unknowns that are solved using a nested solution architecture that employs the gradient-based Newton-Raphson iteration method. Presented issues are related to constitutive model development for a stand-alone viscoplasticity code that introduces concepts related to stress return mapping, which requires stress corrections for the strain driven approach of the radial return method. The same concept is used when solving special case problems in structural analysis, such as plane stress problems. During structural analysis through the finite element method, local and global level numerical iterations are carried out simultaneously through a strain driven approach. In the strain driven approach, all components of strain are prescribed, and stress increments are calculated through local numerical integration for the chosen constitutive model. Global iteration then proceeds by solving the nodal force equilibrium.

In the stand-alone viscoplasticity model, where not all strains components are prescribed, stress correction terms are required. One of the classical examples is a uniaxial strain-controlled loading history. In such case, only one strain increment is prescribed, while the other five components of strain are unknown, but corresponding stress increments are known and equals to zero which is used to modify the numerical scheme according to [4]. A general equation for stress increment of radial return method can be expressed as:

$$\Delta\boldsymbol{\sigma} = \Delta\boldsymbol{\sigma}^{\boldsymbol{tr}} - 2G\Delta\boldsymbol{\varepsilon}_{\boldsymbol{p}}\,, \tag{4}$$

$$\begin{bmatrix}\Delta\sigma_1\\0\\0\end{bmatrix} = \begin{bmatrix}k_1 & k_2 & k_2\\k_2 & k_1 & k_2\\k_2 & k_2 & k_1\end{bmatrix}\begin{bmatrix}\Delta\varepsilon_1\\\Delta\varepsilon_2\\\Delta\varepsilon_3\end{bmatrix} - 2G\begin{bmatrix}\Delta\varepsilon_{p1}\\\Delta\varepsilon_{p2}\\\Delta\varepsilon_{p3}\end{bmatrix}. \tag{5}$$

It should be noted that the elastic matrix is described here by general terms k_1 and k_2. Strain prescriptions $\Delta\varepsilon_2$ and $\Delta\varepsilon_3$ in elastic predictor are not known, hence unknown strain values are taken out and replaced by new trial strain values as follows:

$$\begin{bmatrix}\Delta\sigma_1\\0\\0\end{bmatrix} = \begin{bmatrix}k_1 & k_2 & k_2\\k_2 & k_1 & k_2\\k_2 & k_2 & k_1\end{bmatrix}\begin{bmatrix}\Delta\varepsilon_1\\\Delta\varepsilon_2^{tr}\\\Delta\varepsilon_3^{tr}\end{bmatrix} - 2G\begin{bmatrix}\Delta\varepsilon_{p1}\\\Delta\varepsilon_{p2}\\\Delta\varepsilon_{p3}\end{bmatrix} + \begin{bmatrix}k_2\\k_1\\k_2\end{bmatrix}(\Delta\varepsilon_2 - \Delta\varepsilon_2^{tr}) + \begin{bmatrix}k_2\\k_2\\k_1\end{bmatrix}(\Delta\varepsilon_3 - \Delta\varepsilon_3^{tr})\,. \tag{6}$$

New trial strain values correspond to the pure elastic loading and can be directly obtained from Hooke's law. The third and the fourth correction terms on the right side of Eq. (3) can be modified. By setting $\boldsymbol{q} = \boldsymbol{s} - \boldsymbol{a}$ and $J_q = J(\boldsymbol{s} - \boldsymbol{a})$ increment of plastic deformation can be expressed as

$$\Delta\boldsymbol{\varepsilon}_{\boldsymbol{p}} = \frac{3}{2}\Delta p\frac{\boldsymbol{q}}{J_q}. \tag{7}$$

Stress update equation can be now derived from equations (3) and (4) in deviatoric form as

$$\boldsymbol{s} = \boldsymbol{s}^{\boldsymbol{tr}} - 2G\Delta\boldsymbol{\varepsilon}_{\boldsymbol{p}} + \frac{\Delta p}{J_q}\boldsymbol{s}_{\boldsymbol{cor}}\,. \tag{8}$$

Subtracting backstress from both sides of the equation (5) leads to the final equation (to simplify the equation, basic backstress without static recovery term and mean stress evolution term is used)

$$\boldsymbol{q} = \boldsymbol{s} - \boldsymbol{a} = \frac{\boldsymbol{s}^{\boldsymbol{tr}} - \sum_{i=1}^{N}\frac{\boldsymbol{a}_{\boldsymbol{n,i}}}{1+\gamma_i\Delta p} + \frac{\Delta p}{J_q}\boldsymbol{s}_{\boldsymbol{cor}}}{1 + \frac{\Delta p}{J_q}\left(3G + \sum_{i=1}^{N}\frac{C_i}{1+\gamma_i\Delta p}\right)}\,. \tag{9}$$

The equation (6) is a non-linear equation because the correction term $\boldsymbol{s}_{cor}$ contains unknown $\boldsymbol{q} = \boldsymbol{s} - \boldsymbol{a}$ and is updated iteratively. This means that in the stand-alone code with the correction terms, convergence of $\boldsymbol{q}$ must be checked as well. After the whole numerical procedure is done, unknown strain components can be determined. The rest of the numerical scheme will proceed according to the standard method.

Stand-alone code has been created for this model to simulate the strain-controlled uniaxial fatigue and the stress-controlled uniaxial creep responses. Stand-alone codes have been incorporated into the optimization procedure based on least-squares method to determine material parameters of the model. Results were checked against the finite element method analysis.

3. Conclusion

The modified unified viscoplastic constitutive material model is proposed for simulating the temperature and time-dependent cyclic behaviour of the material SiMo 4.06. This advanced constitutive model is numerically integrated and implemented into a finite element method analysis as well as into the stand-alone code to simulate the strain-controlled uniaxial low-cycle fatigue tests and the uniaxial creep tests. Stand-alone codes have been incorporated into the optimization procedure to determine material parameters of the model.

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