COMPUTATIONAL MECHANICS 2021
36th conference with international participation
Srní
November 8 - 10, 2021

Acoustic streaming of viscous fluid in a confining layer with vibrating walls – numerical simulations

F. Moravcová, E. Rohan

[a] *NTIS – New Technologies for the Information Society, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 8, 301 00 Plzeň, Czech Republic*

Acoustic streaming (AS) is a non-intuitive phenomenon occurring in a high-intensity sound field. In general, AS is a quasi-stationary flow generated by a nonlinear acoustic wave propagating in a viscous fluid. This flow is produced due to inhomogeneities in a viscous fluid due to non-zero divergence of the Reynolds stress (due to the kinetic energy of the acoustic wave), or due to vibrating fluid-solid interface (effects of surface acoustic waves). It is observed at fluid boundary layers as the Rayleigh streaming due thermal and/or viscous phenomena, or in the bulk fluid as the high-frequency Eckart streaming. The so-called micro-streaming in the vicinity of channel walls can provoke cavitation associated with actions of microbubbles (cavitation microstreaming). Mathematical modelling of the AS was originated by Rayleigh (1884). Major pioneering contributions are due to Nyborg and Lighthill [1, 3] who established the fundamental framework for the nonlinear acoustic wave treatment using the perturbation theory.

In the present study, we explore the AS induced by the vibrating fluid-solid interface producing surface acoustic waves. For this, we consider laminar flows in vibrating channels, as shown in Fig. 1. A viscous barotropic fluid is considered, such that the adiabatic condition holds.

By pursuing the standard perturbation analysis, the flow field variables are decomposed into time-periodic components, representing the primary acoustic response, and the components representing the secondary quasi-stationary effects which can describe the acoustic streaming phenomenon. Besides the model arising due this perturbation analysis, we apply the standard approach of computational analysis by means of the relevant flow model, thus, providing direct numerical simulations (DNS) of the phenomenon. Open source software OpenFOAM is employed to solve both the types of the evolutionary boundary value problems.

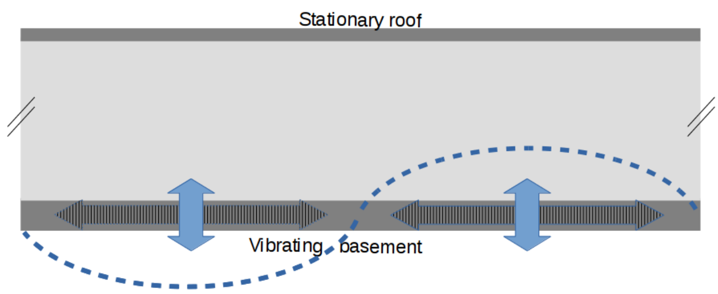
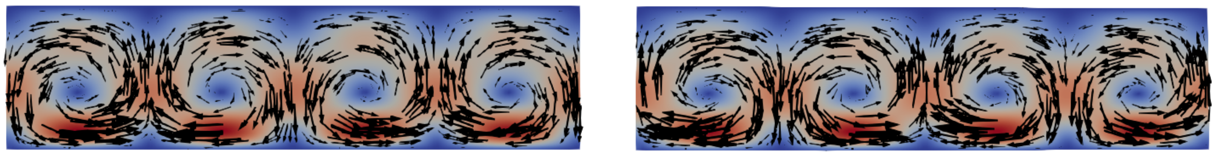


Fig. 1. Flow domain and boundaries conditions



$\boldsymbol{U} \parallel \boldsymbol{n}$ $\boldsymbol{U} \cdot \boldsymbol{n} = 0$

Fig. 2. Acoustic streaming due to vibrating wall Γ_w with normal (left) $U_1 = 0$, and tangential (right) $U_2 = 0$ displacement amplitude, see (3). Note the different vortex orientations

Flow in a confined 2D layer with vibrating walls We consider a 2D thin slab $\Omega =]0, L[\times]0, b[\subset \mathbb{R}^2$ representing a section of the infinite layer $]-\infty, +\infty[\times]0, b[$ occupied by a viscous barotropic fluid. Domain Ω is bounded by $\partial\Omega$ consisting of four parts Γ_w, Γ_0 and $\Gamma_\#$. The flow is induced by harmonic oscillations of the inferior wall $\Gamma_w = \{x \in \partial\Omega | x_2 = 0\}$, whereas fixed superior wall $\Gamma_0 = \{x \in \partial\Omega | x_2 = b\}$ is considered, cf. [2]. For practical reasons of the numerical simulations, periodic conditions are prescribed on the vertical boundary segments, $\Gamma_\# = \{x \in \partial\Omega | x_1 = 0, L\}$. The velocity vector $\boldsymbol{u}$, the density ρ and the pressure p satisfy the Navier-Stokes equations involving the mass and momentum conservation equations,

$$\begin{aligned} \partial_t \rho + \nabla.(\rho \boldsymbol{u}) &= 0 \;, \\ \partial_t(\rho \boldsymbol{u}) + \nabla.(\rho \boldsymbol{u} \otimes \boldsymbol{u}) &= -\nabla p + \nabla \cdot \boldsymbol{\sigma}^{vi}(\boldsymbol{u}) \;, \end{aligned} \tag{1}$$

where the viscous part of the stress $\boldsymbol{\sigma}^{vi}$ is defined using the viscosity coefficients μ and η. Thus, we may introduce operator $\hat{\mathcal{A}}(\boldsymbol{u})$, as follows

$$\hat{\mathcal{A}}(\boldsymbol{u}) := \nabla \cdot \boldsymbol{\sigma}^{vi}(\boldsymbol{u}) = \mu \nabla^2 \boldsymbol{u} + (\eta + \frac{1}{3}\mu)\nabla(\nabla \cdot \boldsymbol{u}) \;. \tag{2}$$

Besides the barotropic fluid, we may consider an incompressible fluid which yields $\hat{\mathcal{A}}(\boldsymbol{u}) = \mu \nabla^2 \boldsymbol{u}$.

The vibrations of the wall Γ_w are defined in terms of prescribed fluid velocity $\boldsymbol{v} = \boldsymbol{w}$,

$$\boldsymbol{w}(x, t) = \boldsymbol{U} \cos\left(\frac{2\pi x}{L}\right) \sin\left(\frac{2\pi t}{T}\right) , \tag{3}$$

where $\boldsymbol{U} = (U_1, U_2)$ is a given amplitude and T is the time period.

Although the fluid oscillates with frequency $\omega = 2\pi/T$ in the response to the vibrating wall, we are interested in the behaviour observed at a time scale much larger than the period T. For this, any quantity $q(x, t)$ is averaged to define $\overline{q}(x, t)$,

$$\overline{q}(x, t) := \langle q \rangle := \frac{1}{T} \int_t^{t+T} q(x, \tau) \mathrm{d}\tau \;. \tag{4}$$

Solution methods The flow equations (1) can be either solved directly (the DNS approach) or in a decomposed form [3] obtained due to the expansion of the state variables with respect to a perturbation parameter ϵ, such that

$$\begin{aligned} \boldsymbol{u}(\boldsymbol{x}, t) &= \epsilon \boldsymbol{u}_1(\boldsymbol{x}, t) + \epsilon^2 \boldsymbol{u}_2(\boldsymbol{x}, t) \;, \\ p(\boldsymbol{x}, t) &= p_0 + \epsilon p_1(\boldsymbol{x}, t) + \epsilon^2 p_2(\boldsymbol{x}, t) \;, \\ \rho(\boldsymbol{x}, t) &= \rho_0 + \epsilon \rho_1(\boldsymbol{x}, t) + \epsilon^2 \rho_2(\boldsymbol{x}, t) \;. \end{aligned} \tag{5}$$

The "zero order" variables labelled by $_0$ are constant in time, whereas the "first order" ones labelled by $_1$ are assumed to be T-periodic in time. Note that the equilibrium velocity is assumed to vanish ($\boldsymbol{u}_0 = 0$) and the equilibrium pressure p_0 and density ρ_0 are constants. Obviously, for an incompressible fluid, $\rho = \rho_0$. In other cases, we assume the wave propagation as an adiabatic process and employ the following barotropic approximation relating the pressure perturbation to the density perturbation through the sound speed c_0, see also [4]. By virtue of (5), the Taylor expansion of the pressure p in response to ρ yields

$$p_1 = c_0^2 \rho_1 \ , \ \ p_2 = c_0^2 \rho_2 + c_0 d_0 \rho_1^2 \ , \quad \text{where } c_0 = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \ , \ \ d_0 := \left(\frac{\partial c_0}{\partial \rho}\right)_s \ . \tag{6}$$

Below the rescaled pressure $\hat{p}_1 := p_1/\rho_0$ is employed.

Upon substituting (5) and (6) in (1) and pursuing the standard split according to orders in ϵ, two problems for the couples $(\hat{p}_1, \boldsymbol{u}_1)$ and $(\hat{p}_2, \boldsymbol{u}_2)$ are identified being governed by the following linear equations (here we consider the barotropic fluid) whereby the averaging (4) is applied,

$$\begin{array}{ll} \partial_t \hat{p}_1 + c_0^2 \nabla \cdot \boldsymbol{u}_1 = 0 \ , & \partial_t \bar{\hat{p}}_2 + c_0^2 \nabla \cdot \bar{\boldsymbol{u}}_2 = N \ , \\ \partial_t \boldsymbol{u}_1 = -\nabla \hat{p}_1 + \hat{\mathcal{A}}(\boldsymbol{u}_1) \ , & \partial_t \bar{\boldsymbol{u}}_2 = \boldsymbol{F} - \nabla \bar{\hat{p}}_2 + \hat{\mathcal{A}}(\bar{\boldsymbol{u}}_2) \ , \end{array} \tag{7}$$

see (2), where N and $\boldsymbol{F}$ are computed, as follows

$$N := -\left\langle \nabla.(\hat{p}_1 \boldsymbol{u}_1) \right\rangle \ , \ \ \boldsymbol{F} := -\left\langle \nabla.(\boldsymbol{u}_1 \otimes \boldsymbol{u}_1) \right\rangle \ , \tag{8}$$

providing the driving forces for the acoustic streaming. For the incompressible case, the continuity equations in (7) are reduced to $\nabla \cdot \boldsymbol{u}_1 = 0$ and $\nabla \cdot \bar{\boldsymbol{u}}_2 = 0$. This decomposed form of the AS problem can be solved using any standard CFD computational tool. For this we employed the OpenFOAM to write our own solvers.

To illustrate the AS phenomenon, in Fig. 2 we depict the flow field $\bar{\boldsymbol{u}}_2$ in domain Ω. The steady state vortices are generated by vibrations of the lower edge in the normal direction, i.e., with an amplitude $\boldsymbol{U} = (0, U_2)$, see (3). Similar pattern is obtained for the tangential vibrations when $\boldsymbol{U} = (U_1, 0)$, whereby small differences between the barotropic and incompressible fluids were observed. In Fig. 3, the relative magnitude $|\bar{\boldsymbol{u}}|/u^*$ of the velocity obtained using the DNS method is displayed along a horizontal and a vertical sections located at $x_2/b = 0, 1/4, 1/2, 3/4$ and $x_1/L = 0, 3/8, 1/4$, respectively, whereby $u^* = \frac{1}{|\Omega|}\int_\Omega \bar{\boldsymbol{u}} d\boldsymbol{x}$. Quite similar curves are obtained while plotting the solution $\bar{\boldsymbol{u}}_2$ of the expanded problem (7). The difference $(|\bar{\boldsymbol{u}}| - |\bar{\boldsymbol{u}}_2|)/u^*$ between the two solutions is less than 5% (Fig. 4).

Remarks and perspectives The computational study reported very briefly in this paper enabled to reveal the Acoustic Streaming (AS) phenomenon in the context of numerical solution methods. Although the phenomenon has been studied and reported in the literature over past decades in various contexts, our intent is to study the AS in periodic scaffolds and to account for thermal and deformation effects which bring a two-way coupling between the first and second-order problems of the decomposed form, see (7).

Acknowledgements

The research has been supported by the grant project GACR 21-16406S of the Czech Science Foundation, and in a part by the European Regional Development Fund-Project "Application of Modern Technologies in Medicine and Industry" (No. CZ.02.1.01/0.0/ 0.0/17 048/0007280) of the Czech Ministry of Education, Youth and Sports.

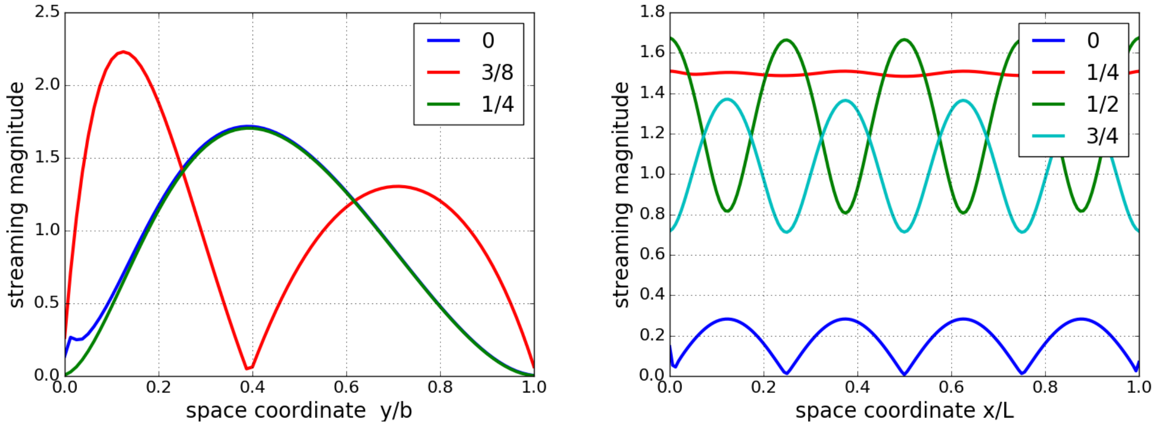


Fig. 3. Velocity magnitude $|\bar{\boldsymbol{u}}|/u^*$ along the vertical (*left*) and horizontal (*right*) line sections

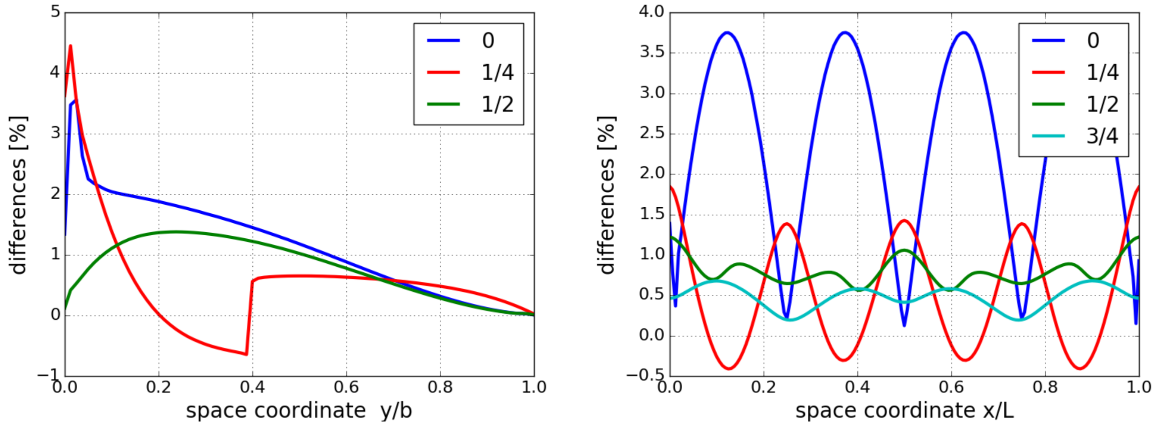


Fig. 4. Relative differences $(|\bar{\boldsymbol{u}}| - |\bar{\boldsymbol{u}_2}|)/u*$ between the direct and expansion resolution in %

References

[1] Lighthill, J., Acoustic streaming, Journal of Sound and Vibration 61 (3) (1978) 391-418.

[2] Loh, B.-G., Hyun, S., Ro, P. I., Kleinstreuer, C., Acoustic streaming induced by ultrasonic flexural vibrations and associated enhancement of convective heat transfer, The Journal of the Acoustical Society of America 111 (2) (2002) 875-883.

[3] Nyborg, W. L., Acoustic streaming due to attenuated plane waves, The Journal of the Acoustical Society of America 25 (1) (1953) 68-75.

[4] Wu, J., Acoustic streaming and its applications, Fluids 3 (4) (2018) No. 108.