COMPUTATIONAL MECHANICS 2021
36th conference with international participation
Srní
November 8 - 10, 2021

Control of modal properties of beam structures using the reinforcing core

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The harmful effects caused by the occurrence of undesired vibrations in machines and various constructions have in many cases fatal consequences for their functionality and in extreme cases also for their structural integrity. In view of the need to eliminate these undesirable states that arise during operation, it is important to create conditions for modifying the modal properties of the relevant structural elements of the mechanical system.

The beam structures can be considered as basic structural elements applied in mechanical and civil structures. A very dangerous state occurs when these structures are exposed to inappropriate dynamic loading effects, which in unfavorable cases can cause their resonant state. One of the important tasks in structural design of the beam structures should be the ability to prevent or reduce the level of unwanted vibrations. The dynamical properties of the beam structures [1], [3], [6] are depended on their structures, geometrical parameters and material properties. The beam structures are usually made of homogeneous material these structures in many cases do not have the required dynamic properties.

Generally, we consider the beam structure (Fig.1) with rectangular cross section (b_0, h_0) and with hole along whole beam length L_0. The moving reinforcing core (cylinder shape) with diameter d_c is inserted into beam hole. The length L_c of inserted cylindrical core is less than the length beam structure ($L_c < L_0$).

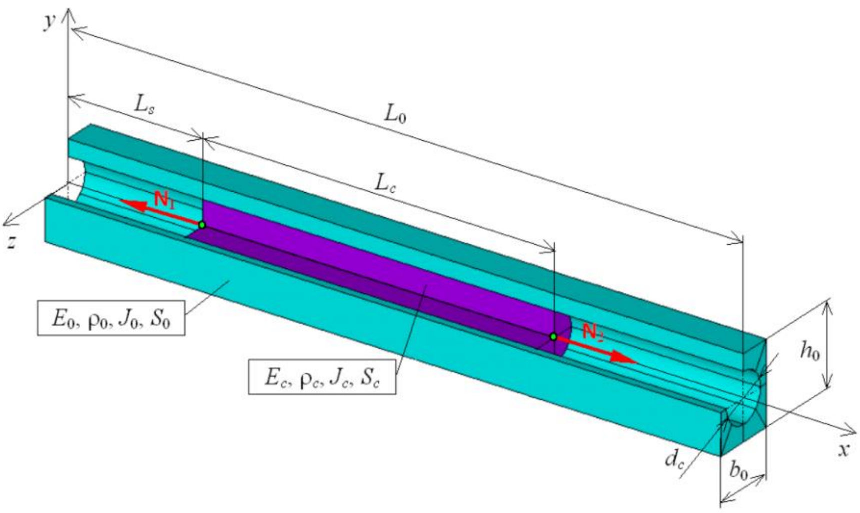

Fig. 1. General model of beam structure with inserted pre-stressed reinforcing core

On both end cross-section surfaces of the reinforcing core, the pre-stressed forces can be applied. Also the position of the inserted reinforcing core can be changed. Then the beam structure has three different structural fields. It is clear [4], [5], that the changing core position, geometric parameters and material properties of reinforcing core and also prestressing forces, the induce the change in the distribution of mass and flexural stiffness of the modified beam structure, which subsequently causes a change in the modal properties.

The following assumptions [5] to the creation of mathematical model of the modified beam structure (Fig. 1) are considered:

- cross-section parts of the beam structure lie in plane perpendicular to the neutral x-axis,
- beam cross-sections before and during beam deformation is assumed as planar,
- cross-sections of beam profile and core are symmetrical with respect to both axes y, z,
- material properties of the beam structure parts are isotropic and homogeneous,
- perfect adhesion at the interface of beam-core is supposed.

The computational model describing the studied beam structure (Fig. 1) is based on the assumptions of the Euler-Bernoulli beam theory. The general equation [2] for bending vibrations of beam with axial pre-stressing forces acting on core, for each j^{th} segment of beam, can be written in the form

$$E_j J_j \frac{\partial^4 w_j(x_j,t)}{\partial x_j^4} - N_j \frac{\partial^2 w_j(x_j,t)}{\partial x_j^2} + \rho_j S_j \frac{\partial^2 w_j(x_j,t)}{\partial t^2} = 0, \qquad (1)$$

where $w_j(x_j, t)$ is vibrating beam deflection.

For each segment in the beam structure, the flexural stiffness E_jJ_j and mass ρ_jS_j of the beam are constant. It must be noted that sign minus before force effect N_j is for a tensile load and sing plus is for a compression load.

By introducing the solution $w_j(x_j,t) = W_j(x_j)T(t)$, the equation (1) has following form

$$\overline{W}_j^{VI}(\xi_j) - \alpha_j^2 \overline{W}_j^{II}(\xi_j) - \beta_j^4 \overline{W}_j(\xi_j) = 0, \qquad (2)$$

where parameters α_j and β_j are axpressed by

$$\alpha_j = \sqrt{\frac{N_j}{E_j J_j} L_0^2} \qquad \text{and} \qquad \beta_j = \sqrt[4]{\frac{\rho_j S_j}{E_j J_j} L_0^4 \omega_{0m}^2} \qquad (3)$$

and ω_{0m} is natural angular frequency modified beam structure. dimensionless parameters are introduced in the form

$$\overline{W}_j(\xi_i) = \frac{W_j(x_j)}{L_0}, \qquad \xi_j = \frac{x_j}{L_0}, \qquad \xi_s = \frac{L_s}{L_0}, \qquad \xi_c = \frac{L_c}{L_0}. \qquad (4)$$

By solving the eigenvalue problem $\lambda_{j,k}^4 - \alpha_j^2 \lambda_{j,k}^2 - \beta_j^4 = 0$ for j^{th} beam segment, the eigenvalues $\lambda_{j,k}$ ($k = 1 \div 4$) are obtained.

The solution of the equation (2), for j^{th} beam segment, has the following form

$$\overline{W}_j(\xi_j) = A_{j1}\cos(\eta_{j1}\xi_j) + A_{j2}\sin(\eta_{j1}\xi_j) + A_{j3}\cosh(\eta_{j2}\xi_j) + A_{j4}\sinh(\eta_{j2}\xi_j), \qquad (5)$$

where $$\eta_{j1} = \sqrt{\frac{1}{2}\left[\sqrt{\alpha_j^4 + 4\beta_j^4} - \alpha_j^2\right]}, \qquad \eta_{j2} = \sqrt{\frac{1}{2}\left[\sqrt{\alpha_j^4 + 4\beta_j^4} + \alpha_j^2\right]} \qquad (6)$$

and $A_{j1} \div A_{j4}$ are integration constants.

The boundary conditions for the clamped-free beam structure and for core position $\xi_s = 0$ are presented in Table 1. The intervals for dimensinless parameters ξ_j (j = 1,2) are defined as follows

$$\xi_1 \in \langle 0,\ \xi_c \rangle, \qquad \xi_2 \in \langle 0,\ 1 - \xi_c \rangle. \qquad (7)$$

Table 1. Boundary conditions for beam structrure

$\xi_1 = 0$	$\overline{W}_1(\xi_1)\big\vert_{\xi_1=0} = 0$
	$\overline{W}_1'(\xi_1)\big\vert_{\xi_1=0} = 0$
$\xi_1 = \xi_c$	$\overline{W}_1(\xi_1)\big\vert_{\xi_1=\xi_c} = \overline{W}_2(\xi_2)\big\vert_{\xi_2=1-\xi_c}$
$\xi_2 = 1 - \xi_c$	$\overline{W}_1'(\xi_1)\big\vert_{\xi_1=\xi_c} = -\overline{W}_2'(\xi_2)\big\vert_{\xi_2=1-\xi_c}$
	$(1+\Delta_{EJ})\overline{W}_1''(\xi_1)\big\vert_{\xi_1=\xi_c} = \overline{W}_2''(\xi_2)\big\vert_{\xi_2=1-\xi_c}$
	$(1+\Delta_{EJ})\overline{W}_1'''(\xi_1)\big\vert_{\xi_1=\xi_c} = -\left\{\overline{W}_2'''(\xi_2) - \overline{N}\,\overline{W}_2'(\xi_2)\right\}\big\vert_{\xi_2=1-\xi_c}$
$\xi_2 = 0$	$\overline{W}_1''(\xi_1)\big\vert_{\xi_2=0} = 0$
	$\overline{W}_2'''(\xi_2)\big\vert_{\xi_2=0} = 0$

$E_1J_1 = E_0J_0(1+\Delta_{EJ})$ $\rho_1S_1 = \rho_0S_0(1+\Delta_{\rho S})$ $N_1 = 0$	$E_2J_2 = E_0J_0$ $\rho_2S_2 = \rho_0S_0$ $N_2 = N$	$\overline{N} = \dfrac{NL_0^2}{E_0J_0}$

The modification parameters

$$\Delta_{EJ} = \frac{E_cJ_c}{E_0J_0} \quad \text{and} \quad \Delta_{\rho S} = \frac{\rho_cS_c}{\rho_0S_0} \tag{8}$$

characterize the proportional changes in the stiffness and mass properties of the beam caused by inserting the core, where E - Young modulus, ρ - density, S - cross-section area, J - second moment of area (subscript 0 is used for the basic beam structure and subscript c is used for the core). Parameter $\overline{N}$ represnts dimensionless axial prestressing force.

Substituting the solution $\overline{W}_j(\xi_j)$ from the equation (5) into the boundary conditions (Table 1), the frequency determinant is created. As a result of the solution of the frequency determinant, the modification function taking into account the changes in the mass and stiffness properties of the modified beam structure, induced by the prestressed core, is expressed. The modification function $f_{m\omega,k}(\overline{N},\xi_c,\Delta_{\rho S},\Delta_{EJ})$ for k^{th} vibration mode shape is defined as the ratio of natural angular frequency $\omega_{0m,k}$ of the modified beam structure to natural angular frequency $\omega_{0,k}$ of the unmodified beam structure

$$\frac{\omega_{0m,k}}{\omega_{0,k}} = f_{m\omega,k}(\overline{N},\xi_c,\Delta_{\rho S},\Delta_{EJ})\,. \tag{9}$$

The modification functions for the first two natural frequencies in dependency on the dimensionless length of core insertion for zero, tensile and compression dimensionless forces and for modification parameters $\Delta_{EJ} = 0.25, \Delta_{\rho S} = 0.25$ are shown in Fig.2. In the case of zero prestressing force, the modification function values are greater than 1.0 over the entire length of core insertion. The values of the first two natural frequencies of the modified beam structure increase proportionally to the length of core insertion. For the first natural frequency the values of modification function are greatest when the core is inserted in the half beam length and the growth is aproximately 10%. When the tensile force is acting at the core end, the pre-stressed state is initialized in the first segment of beam and the first natural frequency of the beam structure increases by 25% when the core is inserted into 90% of the beam length. In the case of compressive force, a decrease in natural frequencies occurs when the core is inserted into an area that exceeds half the length of the beam structure (Fig. 2).

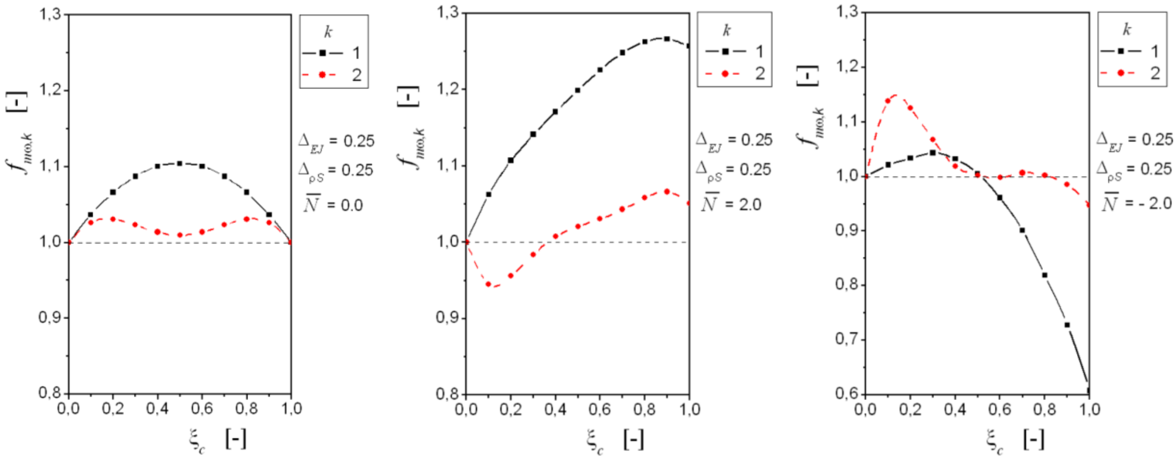

Fig. 2. Dependency of modification functions $f_{m\omega,k}$ (for the first two mode shapes k = 1,2) on core position ξ_c

The technique of structural modification leading to the change of modal properties (natural frequencies, mode shapes) of beam structures, which is based on applications reinforcing core inserted into beam body, its geometrical parametrs and materiál properties and also on prestressing forces acting on the core in the axial direction of the beam structures is presented in ths paper. By changing modification parameters $\overline{N}$, Δ_{EJ}, $\Delta_{\rho S}$, ξ_c is possible to achieve a appropriate modification of natural frequencies of the beam structure (Fig. 1), i.e.

$$\omega_{0m,k} = \omega_{0,k} f_{m\omega,k}(\overline{N}, \xi_c, \Delta_{\rho S}, \Delta_{EJ}) . \qquad (10)$$

Presented beam structure modifications provide the possibilities leading to structural modification, ie. redistribution of beam spatial properties, which could be used to "tune" the modal properties of beam structures to the desired values and thus to eliminate the occurrence of resonant states.

Acknowledgements

The work has been supported by the grant project „EDU-MaTech" Interreg SK-CZ and by the research project KEGA 009STU-4/2021.

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