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On mathematical models of airflow in a glottal channel model periodically closed by flow induced vocal folds vibration

P. Sváček

Czech Technical University in Prague, Faculty of Mechanical Engineering, Dep. of Technical Mathematics, Czech Republic

1. Introduction

In this paper a two-dimensional mathematical model of human voice creation is addressed. Voice production is a complex process involving airflow induced vibrations of (elastic) vocal folds, [10]. The vibrations of the vocal folds generate a sound source, which is modified by the acoustic resonances of vocal tract cavities. The vocal folds oscillations appear at the so-called phonation onset which is basically a flutter type of instability. The phonation onset is characterized by certain airflow rate and a certain prephonatory vocal folds position, see [3, 4]. During the phonation a higher flow rates causes vibrations with increasing amplitudes, which leads to collisions of the vocal folds and to the closure of the glottis.

Consequently, the mathematical modelling of phonation process is challenging task: it needs to address the flow field, the structural deformation as well as acoustics, see e.g. [6, 11]. As the fundamental sound source is significantly influenced by the vocal folds contacts, this phenomena needs to be addressed properly in the model. This is rather difficult as it needs to be included not only to the structural model as impact forces but also to the fluid model, where it influences the geometrical domain deformation with possible topological changes as well as the artificial boundary conditions.

As the airflow velocity in the human glottal region is lower than 100 m/s, the air flow is modelled by the incompressible Navier-Stokes equations. The acoustic part can be modelled by an acoustic model, [9]. The addressed problem is a problem of fluid-structure interaction, where a simplified model of the elastic structure is used, [3, 5]. The attention is paid to the problem of glottis closure. This is realized by a modification of the computational domain, use of an artificial porous media subdomain and a suitable modification of the inlet boundary condition. The described mathematical model is discretized with the aid of the stabilized finite element method and numerical results are shown.

2. Mathematical model

First, we consider a simplified two dimensional model of the computational domain during the phonation onset phase, i.e., in the case of vocal folds are vibrating with only small amplitudes. The two-dimensional computational domain Ω_t for the fluid flow is considered as shown in Fig. 1. In this case the boundary of the computational domain consists of fixed or deformable walls denoted as Γ_{Wt} (it corresponds to the vibrating vocal folds with its upper $\Gamma_{Wt,up}$ and lower part $\Gamma_{Wt,down}$ as well as the other fixed walls Γ_{Wf} and the inlet Γ_I and the outlet Γ_O part of the boundary.

The air flow in the computational domain Ω_t is modelled as incompressible fluid flow described by the system of the incompressible Navier-Stokes equations (cf. [2]) written in the ALE form (cf. [7])

$$\frac{D^{\mathcal{A}}\boldsymbol{u}}{Dt} + ((\boldsymbol{u}-\boldsymbol{w}_D)\cdot\nabla)\boldsymbol{u} = \operatorname{div}\boldsymbol{\tau}^f,$$
$$\nabla\cdot\boldsymbol{u} = 0, \tag{1}$$

where $\boldsymbol{u} = (v_1, v_2)$ is the vector of fluid velocity, $\boldsymbol{\tau}^f = (\tau^f_{ij})$ is the fluid stress tensor given as $\boldsymbol{\tau}^f = -p\mathbb{I}+2\nu\boldsymbol{D}$, $\boldsymbol{D}$ is the symmetric gradient tensor $\boldsymbol{D}(\boldsymbol{u}) = \frac{1}{2}(\nabla\boldsymbol{u}+\nabla^T\boldsymbol{u})$ with components $d_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$, p denotes the kinematic pressure (i.e. the pressure divided byt the constant fluid density) and $\nu > 0$ is the constant kinematic fluid viscosity. In order to take into account the deformation of the computational domain Ω_t the Arbitrary Lagrangian-Eulerian method is used. In this case by $\mathcal{A}_t$ the ALE mapping is denoted, which maps the reference configuration $\Omega_{ref} = \Omega_0$ onto the current configuration Ω_t at any time $t \in [0, T]$, $\boldsymbol{w}_D$ denotes the domain velocity (i.e. the velocity of the point with a fixed reference), and $\frac{D^{\mathcal{A}}\boldsymbol{u}}{Dt}$ is the ALE derivative, i.e. the derivative with respect to the reference configuration Ω_{ref}.

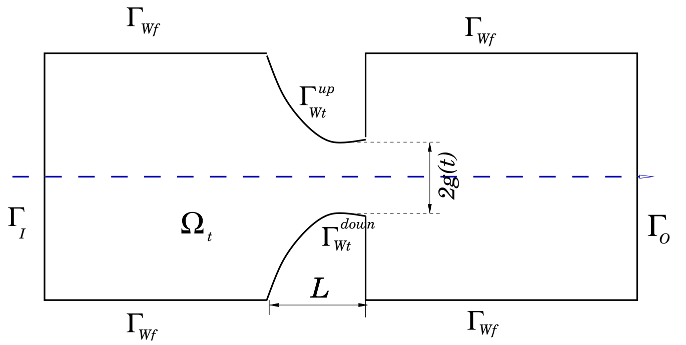

Fig. 1. The computational domain Ω_t with the boundary parts representing the (deformable and undeformable) walls, the inlet and the outlet. The domain is shown in a symmetric configuration with the dashed line showing the axis of symmetry

At the boundary $\partial\Omega^f_t$ of the computational domain formed by mutually disjoint parts $\partial\Omega^f_t = \Gamma_I \cup \Gamma_O \cup \Gamma_{Wt}$, the following boundary conditions are prescribed:

a) $\boldsymbol{u} = \boldsymbol{w}_D$ on Γ_{Wt},

b) $\frac{1}{2}(\boldsymbol{u}\cdot\boldsymbol{n})^-\boldsymbol{u} - \boldsymbol{n}\cdot\boldsymbol{\tau}^f = \frac{1}{\varepsilon}(\boldsymbol{u}-\boldsymbol{u}_I)$ on Γ_I,

v) $\frac{1}{2}(\boldsymbol{u}\cdot\boldsymbol{n})^-\boldsymbol{u} - \boldsymbol{n}\cdot\boldsymbol{\tau}^f = p_{ref}\boldsymbol{n}$ on Γ_O,

where $\boldsymbol{n}$ denotes the unit outward normal vector to $\partial\Omega^f_t$, $\boldsymbol{u}_I$ is a prescribed inlet velocity, p_{ref} is a reference pressure value ($p_{ref} = 0$ in what follows), $\varepsilon > 0$ is a penalization parameter and α^- denotes the negative part of a real number α. Let us emphasize that at Γ_{Wf} the domain velocity equals zero $\boldsymbol{w}_D = 0$, whereas at the moving wall Γ_{Wt} the domain velocity $\boldsymbol{w}_D$ is equal to the velocity of the surface. The boundary condition (2c) weakly imposes the Dirichlet boundary condition $\boldsymbol{u} = \boldsymbol{u}_I$ using the penalization parameter $\varepsilon > 0$.

2.1 Vocal fold vibrations

The elastic deformation of the vibrating vocal fold is described using a simplified model with two degrees of freedom, i.e. the displacements $w_1(t)$ and $w_2(t)$ of the masses m_1 and m_2, respectively (see Fig. 2). These displacements are governed by the equation of motion (see [3] for details)

$$\mathbb{M}\ddot{\boldsymbol{w}} + \mathbb{B}\dot{\boldsymbol{w}} + \mathbb{K}\boldsymbol{w} = -\boldsymbol{F}, \tag{2}$$

where $\mathbb{M}$ is the mass matrix of the system, $\mathbb{K} = \mathrm{diag}(k_1, k_2, k_4, k_5)$ is the diagonal stiffness matrix of the system characterized by spring constants k_1, k_2, and $\mathbb{B} = \varepsilon_1\mathbb{M} + \varepsilon_2\mathbb{K}$ is the matrix of the proportional structural damping, ε_1, ε_2 are the constants of the proportional damping. The mass matrix is given by

$$\mathbb{M} = \begin{pmatrix} m_1 + \frac{m_3}{4} & \frac{m_3}{4} & 0 & 0 \\ \frac{m_3}{4} & m_2 + \frac{m_3}{4} & 0 & 0 \\ 0 & 0 & m_4 + \frac{m_6}{4} & \frac{m_6}{4} \\ 0 & 0 & \frac{m_6}{4} & m_5 + \frac{m_6}{4} \end{pmatrix}, \tag{3}$$

where m_1, m_2, m_3 are the three masses shown in Fig. 2 (left) for the lower part of the vocal fold model and m_4, m_5, m_6 are the three masses shown in Fig. 2 (right) for the upper part of the vocal fold model. The vector $\boldsymbol{F} = \boldsymbol{F}_{imp} + \boldsymbol{F}_{aero}$ consists of the impact forces $\boldsymbol{F}_{imp}$ and the aerodynamical forces $\boldsymbol{F}_{aero} = (F_1, F_2, F_4, F_5)^T$.

The aerodynamical forces are evaluated as surface integrals from the aerodynamical quantites computed in the fluid part of the model, i.e. by the (kinematic) pressure p and by the gradient of the flow velocity $\boldsymbol{u} = (u_1, u_2)$. The displacement of the surface Γ_{Wt}^{down} is determined in terms of w_1, w_2, and it is used as boundary condition for the displacement of any point of the computational domain Ω_0^{ref} onto the domain Ω_t. Similarly the displacement of the surface Γ_{Wt}^{up} is determined in terms of the displacements w_4, w_5.

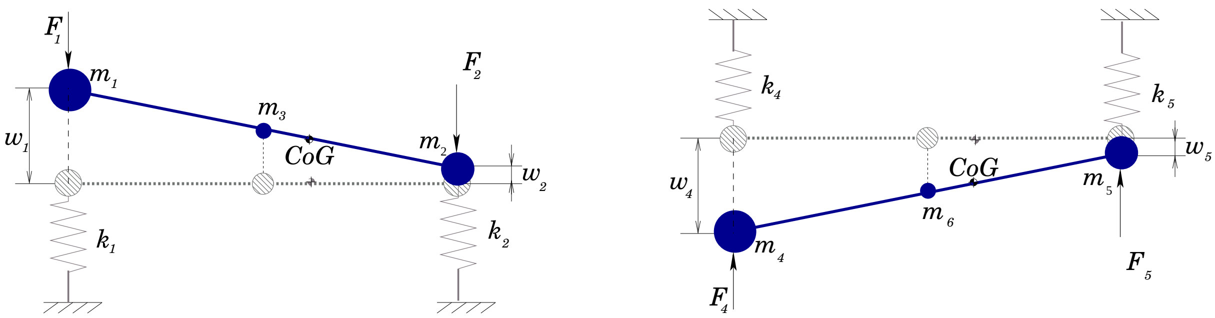

Fig. 2. The aeroelastic model of the down part (*left*) and the upper part (*right*) of the vocal fold

2.2 Contact problem

In order to model the contact of the vocal folds the flow of the air is approximated on a modification of the computational domain still denoted by Ω_t, which is formed of the original fluid domain Ω_t^f and of the part of the domain Ω_t^P, which is the part of the domain Ω_t assumed to be occupied by vocal folds. In practical problem it is realized by determination of the gap $2g(t)$ between the two vocal folds. In case when $g(t) \geq g_{min} > 0$ no modification of the computational domain is needed (here g_{min} a suitable minimal gap threshold). For the case $g(t) \leq g_{min}$, the displacements of the boundary parts Γ_{Wt}^{up} and Γ_{Wt}^{down} is modified, which leads to artificially created fictitious porous media domain Ω_t^p. The flow in Ω_t^P is then governed by the modified equations

$$\frac{D^A\boldsymbol{u}}{Dt} + ((\boldsymbol{u} - \boldsymbol{w}_D) \cdot \nabla)\boldsymbol{u} + \sigma_P\boldsymbol{u} = \mathrm{div}\,\boldsymbol{\tau}^f,$$

where the coefficient σ_P corresponds to the artificial permeability P of the fictitious porous media. This approach can be also understand as penalization, see [1].

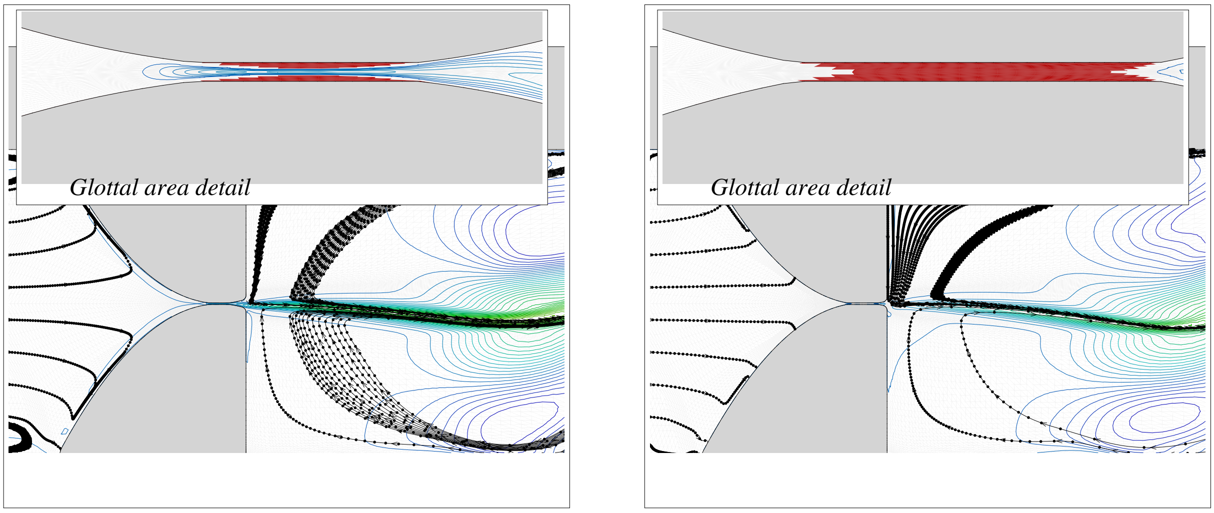

Fig. 3. The flow velocity distribution in the computational domain during two time instants for the case corresponding to post-flutter simulation from [8]

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