Recursive Sine Wave Digital Oscillator with New Method Used for Amplitude Control

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***Abstract* — This work describes the control of the amplitude of the output signal (or amplitude stabilization) in digital sine recursive oscillators. As in the case of analog oscillators, it is also necessary to control the amplitude of digital oscillators with a sine wave. If the amplitude is not controlled, exponential increase or decay of the oscillation amplitude occurred, even if floating-point arithmetic is used. An example of digital sine recursive oscillator control with quadrature outputs, simulation results and design results are given in detail. Oscillator amplitude control is performed in a completely new way with a simple and fast algorithm for finding the maximum (minimum) and amplitude correction in case the amplitude differs from the desired value. The advantage is that the oscillator can be simply implemented by a program in the microcontroller.**

***Keywords* — amplitude control, frequency control, integer arithmetic, quadrature outputs, recursive oscillator, simulation**

I. Introduction

Digital recursive sine oscillators with quadrature outputs (DRSQO), or even with multiphase outputs are often used in signal processing [1-6]. Although it is possible to use oscillators based on the principle of direct digital frequency synthesis (DDS), DRSQOs are often used for their simplicity and the ability to implement them in a microcontroller. However, finite number length, or rounding, leads to instability in the amplitude of the output signal. This article briefly describes some types of DRSQO. The basic equation of simple, direct form of digital oscillator or "biquad" is based on second order digital resonator

$$y(n) = 2r \cdot cos\left(\theta\right) y(n-1) - r^2 y(n-2) \qquad (1)$$

The Z-transform of (1) is

$$Y(z) - 2r \cdot cos\left(\theta\right) Y(z) z^{-1} + r^2 Y(z) z^{-2} = 0 \qquad (2)$$

Characteristic equation is

$$z^2 - 2r \cdot cos\left(\theta\right) z + r^2 = 0 \qquad (3)$$

where θ is angle step per iteration in radians per sample. The next equation determine real digital oscillator output frequency f_O (for iteration sampling frequency f_S)

$$f_O = \theta \cdot f_S / \left(2\pi\right) \qquad \text{[rad, Hz]} \qquad (4)$$

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The absolute values of the roots lie on the unit circle for r=1 (inside of unit circle for r<1 - which causes the amplitude to decrease or outside of unit circle for r>1 - which causes an increase in amplitude). The equations for an oscillator with 2 output signals with mutual phase shift are

$$x(n+1) = cx(n) + dy(n); \quad y(n+1) = ex(n) + fy(n) \qquad (5)$$

After some mathematical manipulation, the difference equation of 2nd order DRSQO is derived

$$y(n+2) - \left(c+f\right) y(n+1) + \left(cf - de\right) y(n) = 0 \qquad (6)$$

For complex roots with an absolute value of 1 and also for complex roots must apply

$$\left|c+f\right| < 2; \quad cf - de = 1 \qquad (7)$$

which represent the oscillation conditions for discrete oscillators, the so-called "discrete equivalent of the Barkhausen criterion".

If c=f then outputs mutual phase shift between the outputs is π/2 therefore it is DRSQO with quadrature outputs. If d=–e both outputs have the same amplitude [7, 8].

This section provides an list of some known DRSQOs. For detailed information see [9-14].

Magic circle oscillator. Properties: Equal amplitudes and a phase difference of 90 degrees plus half a sample.

Staggered oscillator. Properties: Phase difference of 90 degrees but different amplitudes.

Reinsch oscillator. Different amplitudes and a phase difference of 90 degrees plus half a sample

Waveguide oscillator. Phase difference of 90 degrees but different amplitudes.

II. Digital Oscillator with Amplitude Stabilization

Usually, the coupled form DRSQO is defined by equations using floating-point numbers

$$x(n+1) = k_1 x(n) - k_2 y(n); \quad y(n+1) = k_2 x(n) + k_1 y(n) \qquad (8)$$

where k_1=$cos(\theta)$ and k_2=$sin(\theta)$ or $k_2 = \sqrt{1-k_1^2}$. Features of this oscillator: Same amplitudes of output signals and phase difference of 90 degrees. Fig. 1 is a block diagram of an oscillator with rounding blocks and auxiliary blocks for amplitude stabilization. An important part for stabilizing the amplitude of output signals are shift registers s1 and s2 for 3 numbers of the signed integer type.

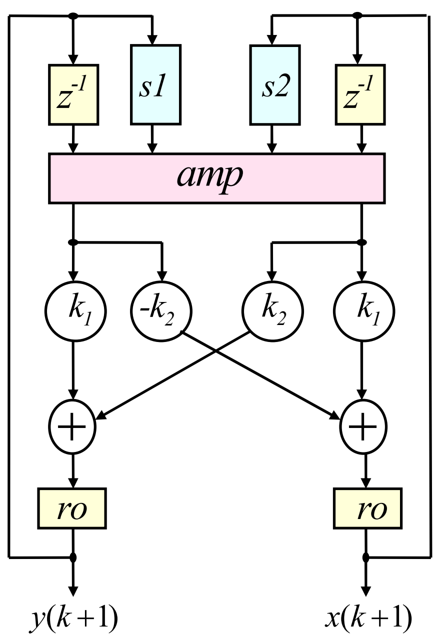

Fig. 1. Block diagram of a digital recursive sine oscillator with qvadrature outputs. *z*-delay block, *k*-constant, *s*- shift register for 3 signed integer numbers, *amp* - amplitude adjustment block, *ro* - rounding block for converting a float number to a signed integer number

The equations of the oscillator according to Fig. 1 with rounding are adjusted to the shape

$$x(n+1) = round\left(k_1 x(n) - k_2 y(n)\right)$$
$$y(n+1) = round\left(k_2 x(n) + k_1 y(n)\right) \qquad (9)$$

where *H=round(G)* rounds the floating point number *G* to the nearest integers. The round function is must be calculated (in C language)

```
if (G>=0) {
H=(int) (G+0.5);
}
else if (G<0) {
H=(int) (G-0.5);
}
```

For integer arithmetic, it is also necessary to select the appropriate amplitude of the oscillator output signals. The oscillator was tested different amplitudes (approx from 256 to 16000) and especially for max. amplitude A of ≈±2047, for a 12 bit unipolar digital analog (D/A) converter. For an oscillator with quadrature outputs, amplitude stabilization can be based on a relationship

$$x(n)^2 + y(n)^2 = A^2 \qquad (10)$$

If $abs(A^2\text{-}(x^2\text{-}y^2)) > delta$, where delta is a suitably chosen constant, the amplitude is adjusted by adding or subtracting the correction value in every step. The effects are almost the same for the *fix*(.) and *round*(.) functions. However, the calculation is time consuming. Another method of amplitude control is based on periodic restarts after each period. The number of samples per 1 period is given by the relation

$$Samples\_per\_period = round(2\pi/\theta) \qquad (11)$$

After reaching a predetermined number of samples, according eq. (11), the oscillator is reset with new initial conditions. Amplitude control is very good, fast, stable and accurate for the *fix*(.) and *round*(.) functions, but there are larger frequency errors.

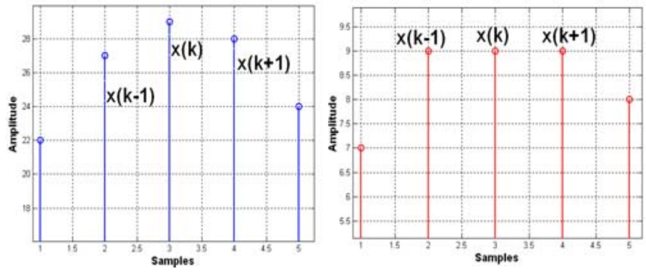

Fig. 2. Example of searching maximum for positive half-wave for different amplitudes and quantization. Left - samples have different sizes, right - samples have the same size

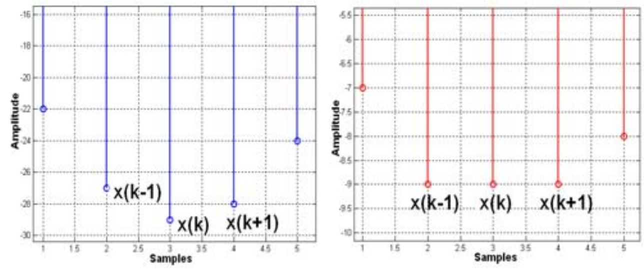

Fig. 3. Example of searching minimum for negative half-wave for different amplitudes and quantization. Left - samples have different sizes, right - samples have the same size

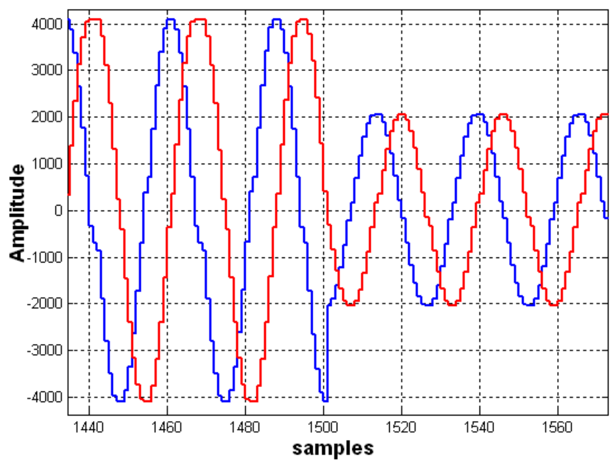

Fig. 4. Example of quadrature oscillator time courses with initial amplitude 4095 and relative frequency 0.257. For sample numbers greater than 1500, the required amplitude is changed to 2047

III. New Method for Amplitude Stabilization

For this reason, the oscillator is supplemented by 2 shift registers *s1* and *s2* (because the oscillator has 2 outputs, see Fig, 1), where the last 3 samples are shifted. Everything is shown in Fig. 2 - for searching for positive maxima and Fig. 3 - for searching for negative minima. For maximum (positive half wave, see Fig. 2)

$$if\ x(k\text{-}1)<x(k)\ \&\ x(k)>x(k+1) \Rightarrow Maximum \qquad (12)$$

It should be noted that for certain levels of quantization and rounding, the values of successive samples may be the same, therefore eq. (12) should be modified to

$$if\ x(k\text{-}1) <= x(k)\ \&\ x(k) >= x(k+1) \Rightarrow Maximum \qquad (13)$$

and for minimum (negative half wave, see Fig. 3)

$$if\ x(k\text{-}1) >= x(k)\ \&\ x(k) <= x(k+1) \Rightarrow Minimum \qquad (14)$$

The amplitude is then corrected (or assigned) to the desired maximum (minimum) amplitude, see Fig. 4.

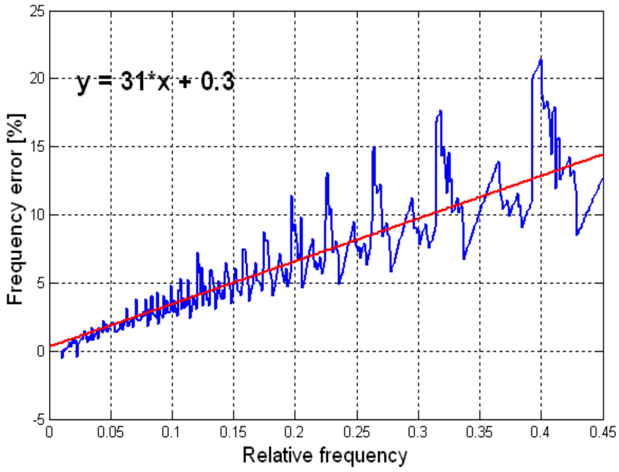

Fig. 5. Frequency errors, without the use of correction for frequency <0.01 to 0.45>. Approximation -red curve and approximation equation

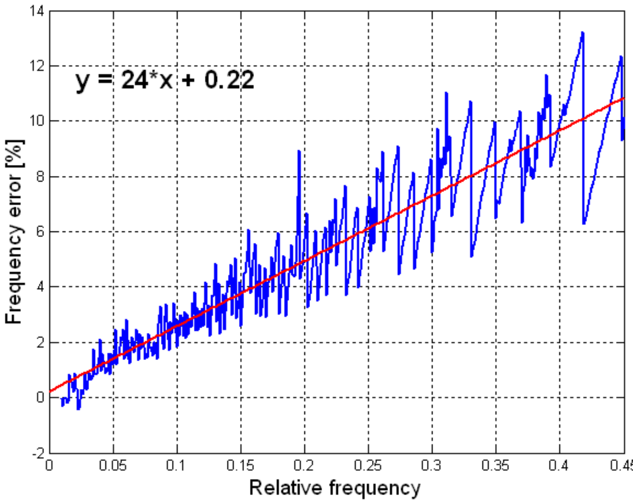

Fig. 6. Frequency errors, with the use of correction for frequency <0.01 to 0.45>. Approximation -red curve and approximation equation

Fig. 4 shows an example of the oscillator time sequences, first for the desired amplitude 4095 and then for 2047. It can be seen from the figure that despite the rounding, the desired amplitudes are met.

It should be noted that rounding also affects the frequency. An oscillator was tested without frequency correction and with frequency correction, which is performed after several periods of the oscillator signal by a small change in k_1 and k_2 if the length of the actual period is different than required (number of samples per period). Fig. 5 shows the result of measuring the% oscillator error without correction. Fig. 6 shows the measurement result in case of correction. The error is calculated according to the formula

$$Error = 100\frac{req\_freq - meas\_freq}{req\_freq}\ [\%] \qquad (15)$$

where *req_freq* is the desired frequency and *meas_freq* is the measured frequency.

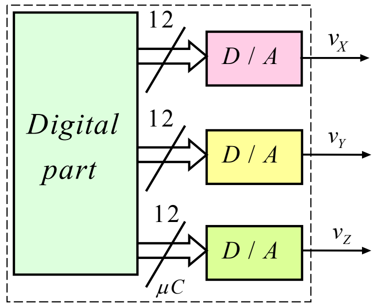

Fig. 7. Block diagram of implemented digital oscillators by microcontroller STM32F303K8

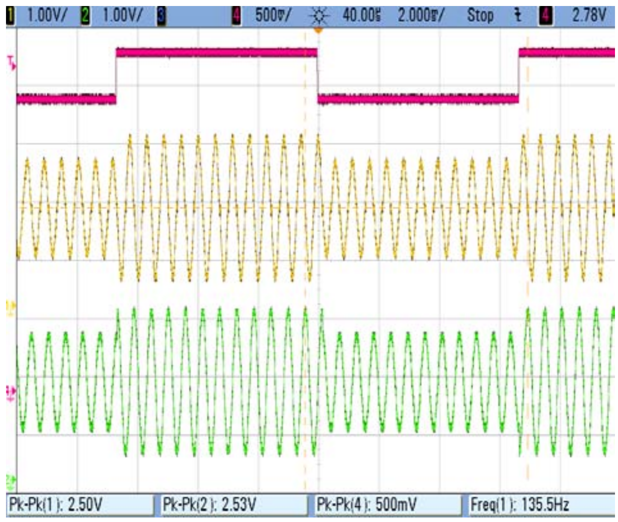

Fig. 8. Oscilloscope record of signals of quadrature oscillator. Example of amplitude control of quadrarure oscillator. Prescribed amplitude value (top), signals *x(k)* and *y(k)* – middle and bottom

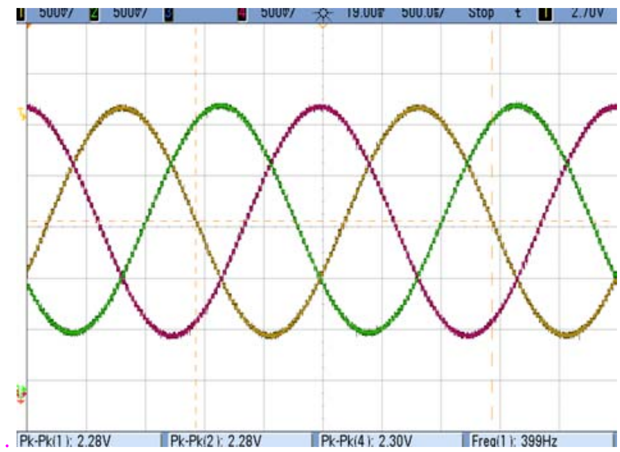

Fig. 9. Oscilloscope record of signals of three phase oscillator

IV. Results

The oscillator was implemented using a microcontroller STM32F303K8 [15], which also contains 3 digital / analog converters (D / A), which could be used to display signals on the oscilloscope and used for measurements. The block diagram of the oscillator is shown in Fig. 7.

Fig..8 is an example of recording signals from an oscilloscope. These are the quadrature oscillator signals at the outputs of the D/A converters. Fig. 9 is an example of output signals of a three-phase sine wave recursive oscillator. The signals were measured at the outputs of D/A converters.

V. Another Application of the Method for Finding the Maximum Amplitude

The described method for finding the maximum / minimum amplitude of the signal can also be used wherever it is necessary to find the local extreme of the signal by a simple and fast digital method.

VI. Conclusion

This paper describes a new method of amplitude stabilization of digital recursive oscillators. The method is based on the comparison of 3 samples of the output signal, which are gradually shifted in the shift register and the maximum or minimum signal is searched. The new method can be used mainly for the output signal rounded to integers. The algorithm is very fast because it only requires comparing the value of integer samples and then adjusting the amplitude according to the desired value.

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