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# Numerical simulations of aeroelastic instabilities in turbine blade cascade by modified Van der Pol model at running excitation

L. Pešek, P. Šnábl, C. S. Prasad, Y. Delanney

*Institute of Thermomechanics, Czech Academy of Sciences, Dolejškova 5, 182 00 Prague, Czech Republic*

Apart from rotary test rig for evaluation of structural dynamics of the bladed wheels, the control flutter experiments have been performed on the linear cascade model in the subsonic wind tunnel in the Institute of Thermomechanics, of CAS, in Prague. These experiments are aimed at stability evaluation of the cascade at running waves or at stability limit testing by flow speed changes or by force impulses of blades. The onset of flutter and its spreading in the cascade are observed, too. The linear cascade model consists of five NACA010 blades. All the blades can be separately excited with electromagnetic torque excitation mechanism and all of them are instrumented to measure the aerodynamic moments which can be used to calculate the aerodynamic work. A more details about the linear blade cascade experimental set up can be found in [3, 8]. To predict a dynamic behaviour in the blade cascade, we have been dealing with simplified theoretical modelling of the aeroelastic instability in turbine blade cascade [2, 4, 5]. Due to the application of the reduced cascade model consisting of simple elements – springs, rigid bodies, linear dampers – and aeroelastic forces introduced by analytical Van der Pol model, it facilitates to study the dangerous states of vibration of such complicated turbine parts [1, 6, 7, 9]. This study is aimed at examination of aeroelastic instabilities of 10-blade cascade at running excitation that arises due to the wakes flowing from stator the blades to the rotating blades. They cause forced excitation in the narrow frequency range.

The computing model of turbine wheel with ten blades with the simplest type of linear connections between neighbouring blades. The sector of blade cascade is shown in Fig. 1.

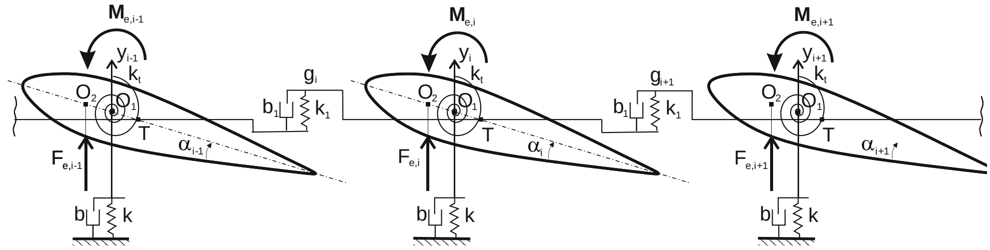


Fig. 1. Section of blade cascade

The blades' interconnections $g_i$ are defined by stiffness $k_1$ and viscous damping $b_1$ constants. These viscous-elastic connections between neighbouring blades can express dynamic properties of connections in turbine disk, blade-shroud or damping wires. The 2 DOF profile has the centre of mass in point *T*. Corresponding moment of inertia is *I*. Flexural axis of this profile is labelled by $O_1$ and the transitional stiffness in vertical direction *y* is *k*. Parallel to the elastic force acts also viscous damping force with coefficient *b*. Pitch spring stiffness around this flexural axis is $k_t$.. This stiffness is again parallel connected by a damping moment with torsional damping coefficient $b_t$. The vertical aerodynamic force $F$ acting on the blade in direction $y$ is shifted in distance $e_2$ into point $O_2$. There is also an aero-elastic moment $M_e = F\,(e_1 + e_2)$ acting around the flexural axis $O_1$ and oriented to increase of pitch angle $\alpha$.

The aerodynamic forces $F$ act on the blades in points $O_2$ in distance of $e_2$=0.005m from the elastic axes $O_1$. The flowing steam through the rotating blade cascade produces besides periodic forced vibration also vertical and torsional aero-elastic self-exciting forces $F_{eV,i}$ and $M_{e,i}$, respectively. Steam flowing through the rotating blade-cascade can cause decrease of damping and aero-elastic flutter instability. Exact mathematical model of this aero-elastic phenomenon is very complicated; therefore we will proposed Van der Pol model [4] which can describe two aerodynamic effects: the first one acting on individual blades controlled by only one blade's motion and the other one, considered here, interacting blades controlled by relative motions of neighbouring blades

$$F_{eV,i} = -\mu_1(1-((y_i - y_{i-1})/r_1)^2)(\dot{y}_i - \dot{y}_{i-1}) + \mu_1(1-((y_{i+1} - y_i)/r_1)^2)(\dot{y}_{i+1} - \dot{y}_i)\,,$$
$$M_{e,i} = -\mu_2(1-((\alpha_i - \alpha_{i-1})/r_2)^2)(\dot{\alpha}_i - \dot{\alpha}_{i-1}) + \mu_2(1-((\alpha_{i+1} - \alpha_i)/r_2)^2)(\dot{\alpha}_{i+1} - \dot{\alpha}_i)\,, \tag{1}$$

where $y_i, \dot{y}_i, \alpha_i, \dot{\alpha}_i$ are vertical and angular displacements of blade $i$ and their velocities, $r_{1,2}$ are displacement limits of blades at which the aerodynamic forces change their sign, $\mu_{1,2}$ give intensities of the considered models.

When the periodic excitation forces and the modified type of Van der Pol forces (1) are applied, differential equations of blade cascade are

$$\begin{aligned} &m\ddot{y}_i + \frac{k_t m e_1}{I}\alpha_i + (k + \frac{k m e_1^2}{I})y_i + b\dot{y}_i + F_{eV,i} + g_i - g_{i+1} = F_{0i}\cos(\omega t - (i-1)\Delta\varphi), \\ &I\ddot{\alpha}_i + k_t\alpha_i + b_t\dot{\alpha}_i - (e_1 + e_2)F_{eV,i} + M_{e,i} + k e_1 y_i = (e_1 + e_2)F_{0i}\cos(\omega t - (i-1)\Delta\varphi), \end{aligned} \quad i = 1,.....,10, \tag{2}$$

where $g_i = k_1(y_i - y_{i-1}) + b_1(\dot{y}_i - \dot{y}_{i-1})$ are viscous-elastic connections among blades. Conditions $g_{11} = g_1,\ F_{eV,11} = F_{eV,1}, M_{e,11} = M_{e,1}$ preserve circular periodicity of the system.

Response curves are computed in the following example for backward running force excitation when $\Delta\varphi = -2\pi/5$ at noozle excitation frequency $\omega = 62.8\,rad/s$. It corresponds to 12 stator blades and revolution frequency 1.25 Hz. The structural profiles parameters

$$m = 0.18\text{ kg},\ k = 50000\text{ kg s}^{-2},\ b = 2\text{ kg s}^{-1},\ I = 0.000025\text{ kg m}^2,\ k_t = 1\text{ kg m}^2\text{ s}^{-2},$$
$$e_1 = e_2 = 0.005\text{ m},\ b_t = 0.00005\text{ kg m}^2\text{ s}^{-2}/\text{rad} \tag{3}$$

and amplitude of external wake force $F_0$ = 0.01 N were applied, too.

As to the intensity factor of Van der Pol model (1) we considered its linear growth over time given by coefficient $c_\mu$. Therefore, we extend the system of differential equations (2) by equation of the first order $\dot{\mu} = c_\mu$ with initial condition $\mu(0) = 5e-4\,kgs^{-1}$ and constant $r = 0.1745\,rad$. The linear growth of intensity factor simulates the increase of instability in flow due to gradually increasing flow speed.

As a simulation case, we choose herein no inter-blades viscous-elastic Kelvin-Voigt connections and only damping connections are via modified van der Pol model of flow aero-elastic forces which corresponds to the tested linear cascade. The time characteristics of the first blade displacements and its aerodynamic moment (Fig. 2) show that flutter arises at time cca 1 s when intensity coefficient achieves a value 5.5e-3. Due to arising self-excited vibrations on the first torsional eigen-frequency the dominant vibration are observed at torsional mode but it causes also increase on vertical displacements. Even after stabilization of vibrations at time 1.5 s the amplitude of vibrations are not constant and course of vibration is non-stationary.

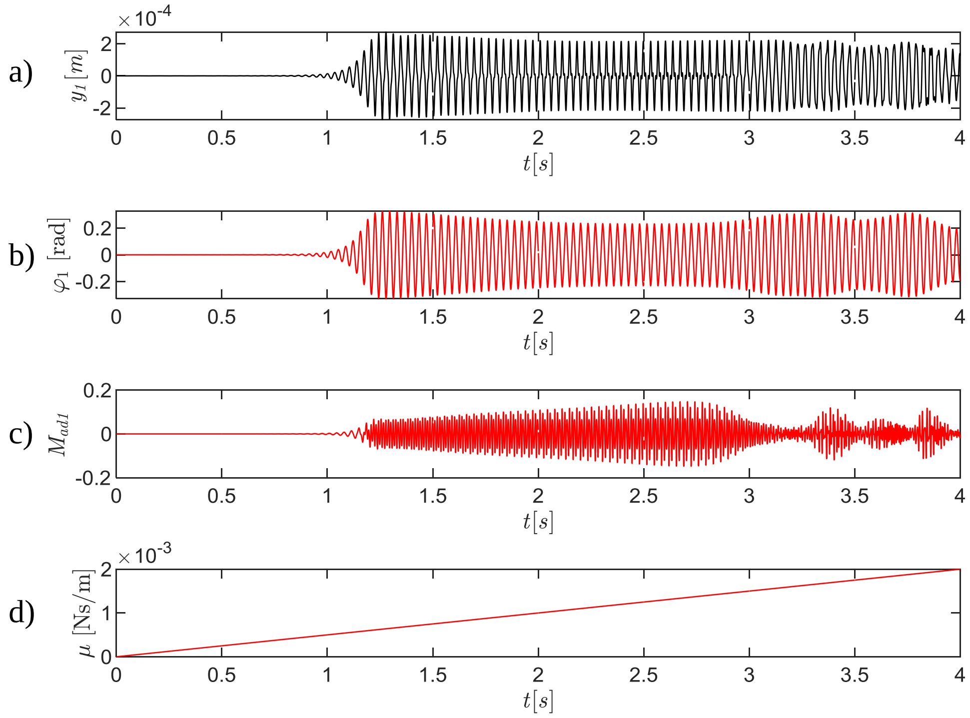


Fig. 2. Time characteristics of amplitudes of vertical and angle displacements (a,b) of 1st blade, of aerodynamic moment of 1st blade (c) and of intensity coefficient (d) at excitation frequency 10 Hz and $\Delta\varphi = -2\pi / 5$

In Fig. 3 we can see mode of vibration across the cascade at certain times: a) at the onset of flutter; b) at the flutter state. It is clear that till the onset of the flutter the mode of vibration has shape of eigenmode with 2 ND and this mode is travelling. However in the state of flutter the vibration mode becomes more complex with higher number of ND. Both these modes are still travelling. However, in longer times (above 3.7 s) when the flutter is more developed, a mode of 4ND prevails at the vibration and this travelling mode becomes standing.

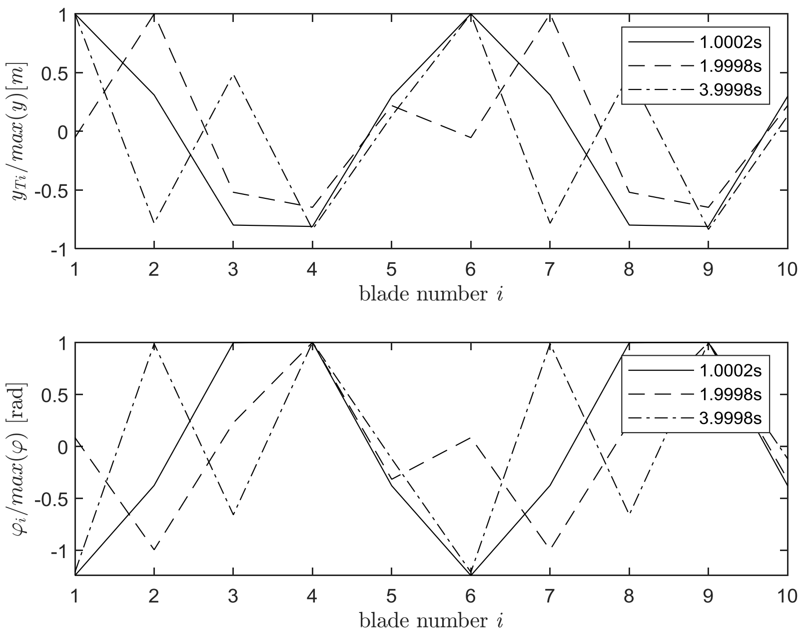


Fig. 3. Modes of vibration of the cascade at times at the beginning (1s) and at at two flutter states (at 2 s and 4 s)

The results of numerical simulations bring valuable findings about dynamic behaviour of the blade cascade of turbine wheels under running nozzle excitation and arising travelling waves at onset and development of the flutter state. The numerical simulations can be further exploited for testing a new algorithm for prediction of the flutter onset detection.


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