COMPUTATIONAL MECHANICS 2022
37th conference with international participation
Srní
November 7 - 9, 2022

Failure probability estimation of functions with binary outcomes via adaptive sequential sampling

M. Vořechovský

Inst. of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Veveří 331/95, 602 00 Brno, Czech Republic

In many computational models for the performance of an engineering product, which feature random input variables, the outcome may be simply binary information: either safe state or failure event. Moreover, computational models sometimes do not return any answer for a combination of inputs because the computation crashes. While the discrete state nature of the computational model outcome is in general acceptable for Monte Carlo type simulation techniques for estimation of failure probability, the need to use computationally expensive models de facto permits this brute force integration. The reason is that the crude Monte Carlo and also various variance reduction techniques such as Importance Sampling for approximation of probabilistic integrals require a high number of simulations to estimate rare event, which is not feasible for models computing one realization more than a few seconds. The samples drawn in sampling-based techniques are in fact independent of each other which is detrimental to the rate of convergence to the true values with an increasing number of outcomes. The use of quasi Monte Carlo techniques is not of much help in reliability problems as the convergence rate improvement is not sufficient.

The absence of any information about the "landscape" in the case of binary performance function also permits the use of methods designed for estimation of failure probability which use the numerical value of the outcome to compute gradients or to estimate the proximity to the failure surface. Even if the performance provides more than just binary output, the state of the system may be a non-smooth or even a discontinuous function defined in the domain of input variables. In these cases, the classical gradient-based methods typically fail. Even if the approximation of failure probability is so effective that it uses only the location and perhaps the shape of the failure surface in the high probability regions (such as FORM or SORM), they still can not be used, because the discovery of the "most probable points" is usualy based on gradient optimization techniques, be it deterministic (design point search in FORM/SORM) or stochastic gradient optimization (subset simulation, etc.).

This paper promotes the recent paper [1] in which a new simple yet efficient algorithm for sequential adaptive selection of points from the input domain of random variables is presented. The extension of the *experimental design* (ED) is designed to automatically balance global *exploration* of new territories in the input space with local *exploitation* of information in regions containing the failure surface, i.e., the boundary between the failure and safe sets. The extension algorithm sequentially adds new points (one by one) in such a way that the new ψ criterion is maximized. The criterion effectively estimates the amount of *probability* being classified by the evaluation of the expensive computational model in a candidate. The evaluation of the criterion is extremely fast as it uses only the known probability density at a candidate and its nearest previously evaluated point and the distance between them. Therefore, it suffices to offer and evaluate the ψ criterion for a large set of candidates, and select the one with the greatest criterion value. Each evaluation of the function in the winning candidate leads to an update of a simple distance-based surrogate model used to censor our candidates with almost sure model output. Only the predefined set of exploratory points and candidates with two different types of outcome in the nearest points are retained for selection. The result of this sequential adaptive selection is a quick refinement of the failure surface or exploration of a new territory, and this is performed proportionally to the gained probability.

At any stage of sequential sampling, the proposed surrogate model can be used to *estimate the failure probability* via tailored importance sampling scheme and a simple distance-based surrogate model. In cases there are more than just one type of failure event, the method can be automatically generalizes to estimate failure probabilities of all even types. Finally, if there is a possibility to use numerical outcomes from the model to build a smooth surrogate, the algorithm can be use it to improve the estimates.

Let us assume the input random vector is bivariate standard Gaussian and the 2D space is divided into safe and failure domains by the wavy circle; see the blue curve in Fig. 1. The left panel in Fig. 1 shows a stage in which the previously evaluated responses are pictured as red (failure) and green (safe) ED points. The panel also shows a large number of candidates which are excluded from further competition for becoming a winning candidate, because their two nearest neighbor ED points are of the same type. The middle panel shows the retained candidates plus the exploration points (empty cirlces) which are blue-colored depending on the proposed ψ criterion. The light blue cross is the winning candidate in which future performance function will be evaluated. The right panel show importance sampling integration nodes.

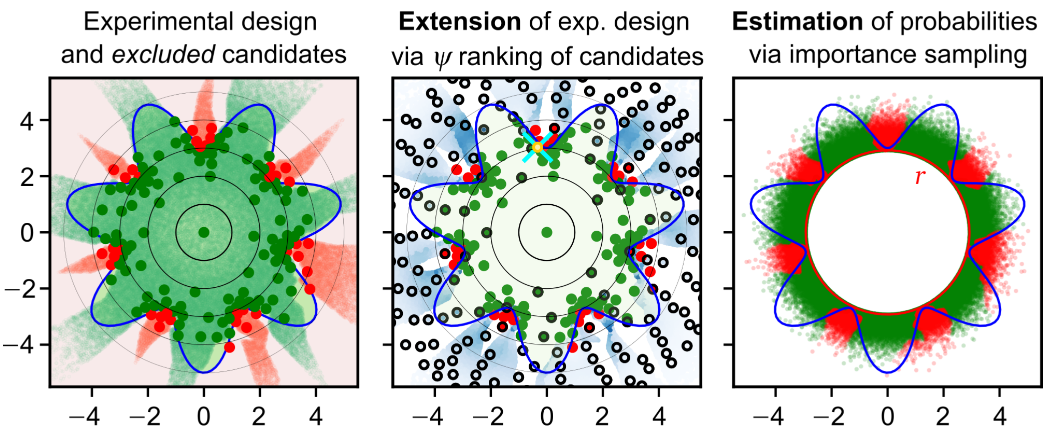

Fig. 1. Illustration of the proposed methodology using a two-dimensional problem with seven effectively disconnected failure domains, each with the same contribution to the probability of failure

Acknowledgement

The work was supported within the framework of research project “Energy dissipation-based approach to stochastic fatigue of concrete considering interacting time and temperature effects” supported by the Czech Science Foundation under project no. 22-06684K.

References

[1] Vořechovský, M., Reliability analysis of discrete-state performance functions via adaptive sequential sampling with detection of failure surfaces, Computer Methods in Applied Mechanics and Engineering, 2022. (in press)
(available via arXiv.org at https://arxiv.org/abs/2208.02475, doi: 10.48550/ARXIV.2208.02475).