

## Study of the oscillation of a pendulum in a magnetic field

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Realization of some technological processes requires application of rotating machines with a vertical rotor mounted in rolling element bearings. One of the requirements put on their operation is minimization of energy losses in the support elements. The losses have several physical reasons. They rise with increasing loading, speed of the shaft rotation, and kinematic viscosity of the lubricant [1]. Replacement of one or more rolling element bearings with a contactless one represents a significant design alteration making it possible to contribute to cuts of energy losses.

Application of magnetic elements for reducing energy losses in the rotor supports was studied at several working places. A historical review and formulas for designing permanent magnetic bearings are reported in [2]. The approach to determination of stiffness of axial bearings using permanent magnets can be found in [3]. The advanced technological solution based on reducing axial loading of rolling element bearings supporting vertical rotors consists in their lifting by means of annular permanent magnets. The details on application of this design solution are reported in [4]. Efficiency of lifting the rotors was analyzed in [5]. A new concept of a shear magnetic bearing was introduced by Zapoměl et al. in [6]. The intentional change of the bearing stiffness to reduce vibration of a rotor system passing through the critical speed was analyzed by Zapoměl and Ferfecki in [7].

The proposed design variant consists in supporting the vertical rotor by one rolling element bearing placed at its upper part and by one axial magnetic bearing mounted in its lower end. The magnetic bearing is composed of an electric coil coupled with the stationary part and of a permanent magnet attached to the rotating part. The magnetic force attracts the permanent magnet, which reduces radial displacement of the lower end of the rotor. The magnetic field between the magnets represents a force coupling between the rotor and the stationary part, which shows some stiffness and affects the system natural frequencies. The controlled change of the stiffness makes it possible to reduce the rotor lateral oscillations in the resonance area.

Applicability and properties of the proposed design concept was studied by means of analysis of oscillations of a pendulum focusing on construction of the frequency response characteristic of the system for variable magnitude of the applied current.

The investigated pendulum (Fig. 1) is coupled with the frame by a revolute joint at its upper end. The permanent magnet is attached to its lower end. The electric coil is coupled with the stationary part and placed under the pendulum. The system is excited by a moment of harmonic time history, which sets the pendulum into a seesaw motion. The task is to construct the frequency response characteristic in dependence on magnitude of the applied current passing the winding of the coil. Amplitude of the oscillations is assumed to be small. The computational simulations should be used to solve the problem.

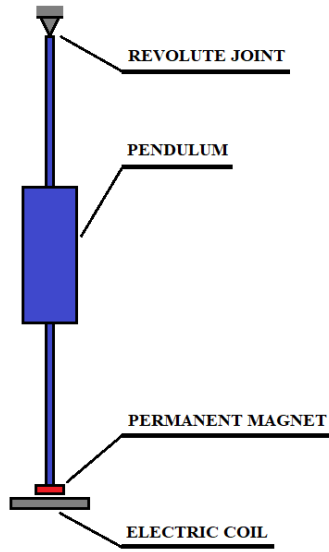


Fig. 1. The investigated system

The introduced coordinate system and the generalized coordinate  $\varphi$  of the pendulum angular position is evident from Fig. 2.

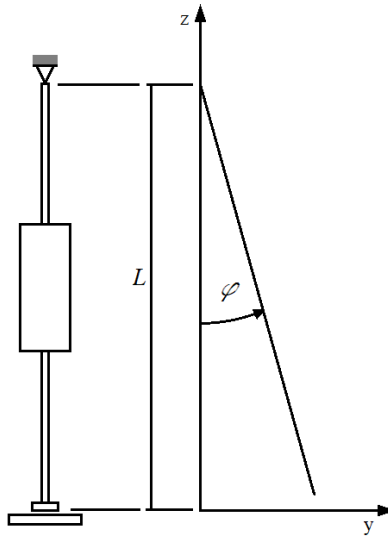


Fig. 2. The introduced coordinate system

In the computational model the pendulum is considered as absolute rigid, the revolute joint as absolute rigid and neutral (with no resistances against motion), and the damping produced by the environment as linear. The coil attached to the stationary part has only one turn. The permanent magnet connected to the pendulum is represented by a magnetic dipole. Because of small displacements, it was assumed that the magnetic field produced by the coil near to its center is homogeneous in the radial direction.

The oscillation of the pendulum is governed by the motion equation

$$J\ddot{\varphi} + b_p\dot{\varphi} + [mgL_T + F_{mag}(I)L]\varphi + M_{mag}(I) = M_A\sin(\omega_B t), \quad (1)$$

$J$  is the pendulum moment of inertia relative to the axis of rotation,  $m$  is the pendulum mass,  $b_P$  is the coefficient of the pendulum external damping,  $L$  is the length of the pendulum,  $L_T$  is the distance between the pendulum axis of rotation and the center of gravity,  $g$  is the gravity acceleration,  $F_{mag}$  is the magnetic force acting on the pendulum,  $M_{mag}$  is the magnetic moment acting on the pendulum,  $I$  is the applied current feeding the coil,  $M_A$  is amplitude of the excitation moment,  $\omega_B$  is the excitation angular frequency,  $t$  is the time,  $\varphi$  is the generalized coordinate of the pendulum position, and  $(\dot{\cdot})$ ,  $(\ddot{\cdot})$  denote the first and second derivatives with respect to time.

The magnetic force and the magnetic moment are functions of the applied current  $I$  feeding the coil. If the current is constant, the motion equation is an ordinary linear differential equation of the second order with constant coefficients

$$J\ddot{\varphi} + b_P\dot{\varphi} + k_e(I)\varphi = M_A\sin(\omega_B t), \quad (2)$$

because the magnetic moment  $M_{mag}$  is proportional to the angle of rotation  $\varphi$ .  $k_e$  is the equivalent stiffness of the system.

The motion equation shows that the natural frequency of the system can be controlled by increasing or decreasing magnitude of the magnetic force and moment.

The principal physical and geometric parameters of the analyzed pendulum are: mass 65 g, moment of inertia 0.0016 kgm<sup>2</sup>, length 305 mm, the gap between the permanent magnet and the electric coil 0.5 mm, magnetic moment of the permanent magnet 1.26 Am<sup>2</sup>, and amplitude of the excitation moment 0.020 Nm.

Application of the current of 200 A rises the natural frequency from 7.15 rad/s to 7.63 rad/s, which corresponds with rising the critical speed as evident from the frequency response characteristic depicted in Fig. 3.

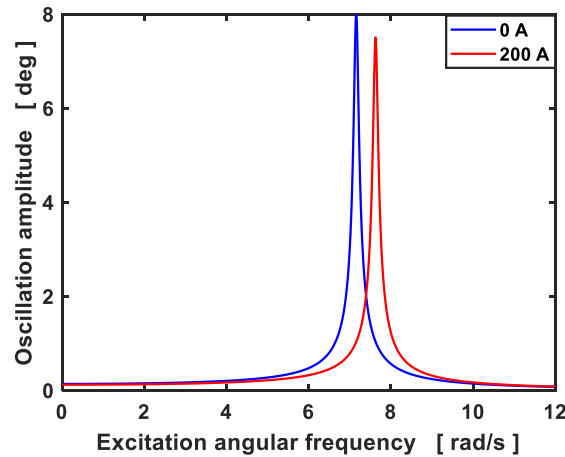


Fig. 3. Frequency response characteristic

The simulation results show that application of the current in the area of sub-critical frequencies and switching the current off in the region of ultra-critical ones makes it possible to reduce amplitude of the pendulum oscillations. This manipulation can be utilized for minimizing the vibration amplitude if the excitation frequency rises and the system passes through the resonance region.

## Acknowledgements

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