Estimation of Eddy Current Losses in SPMSM Based on Harmonic Decomposition

J. Dražan, J. Laksar

***Abstract* – Algorithm for evaluating eddy current losses in surface-mounted permanent magnets situated in synchronous machine is discussed in this paper. Whereas variable air gap permeance causing a ripple of magnetic flux density produced by permanent magnets is considered the main source of these losses. The key objective of this project is to create a fast and relatively accurate tool for setting up the average value of losses in permanent magnets at the prime stage of the electromagnetic design process of such a machine. The method is based on the harmonic decomposition of variable quantities, described by the Fourier series's analytical formulae. First, evaluation is performed for a specified number of discretely organized layers in the permanent magnet. Afterwards, the final result is obtained from the numerical integration of these particular solutions. The algorithm's efficiency is compared with finite element analysis (FEA) in the last part of this paper.**

***Index Terms*—eddy current losses; no-load state; permanent magnet; slotting effect; spatial harmonics; SPMSM; surface mounted permanent magnet synchronous machine.**

I. Introduction

THE amount of electrical devices that help people worldwide in numerous branches of their lives increases every day. However, this market segment attracts more and more companies that develop these devices. Therefore, it creates a competition in which only well and fast, designed products can win.

The following paragraphs describe one of those tools which could help developers during the design process of electric machines. It should speed up the initial electromagnetic design and make it more efficient and precise before finite element analysis (FEA) and prototyping is included into the process. This tool is developed through a computational algorithm in Matlab script. The main task of the algorithm is to evaluate eddy current (EC) losses in surface-mounted permanent magnets of a synchronous machine (SPMSM) at a no-load state.

Temporal variation of magnetic flux density in permanent magnets (PM) causes induction of ECs into their volume. There exist two fundamental origins which influence this magnetic flux density.

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The first one is the magnetic field produced by stator winding. This field consists of a broad spectrum of temporal and spatial higher harmonics besides the fundamental one. Nonzero relative speed exists between these higher harmonics and PMs and causes the induction of eddy currents into them [1], [2].

The second effect influencing the instantaneous value of magnetic flux density in a permanent magnet is the permeance of air gap between stator and rotor. The permeance is not constant along the machine's circumference because of slot openings [3], [4]. A rotating magnet mounted on the rotor produces temporally variable magnetic field in the magnet, causing eddy currents in it [5]. This field can be also decomposed into a spectrum of space harmonics by FFT. This property plays a crucial role in the created algorithm. As mentioned above, the developed algorithm describes the machine's no-load state. Therefore, only the slotting effect is considered a source of ECs.

II. Assumptions and Simplification of the Solution

A 2D mathematical model of electromagnetic field in the machine is used for this purpose. Following considerations have been applied to simplify the model:

- Only radial component of magnetic flux density $B_{\mathrm{PM}}^{\mathrm{R}}$ in permanent magnet is considered,
- Only axial component of eddy current density in permanent magnet is considered $J_{\mathrm{PM}}^{\mathrm{Z}}$,
- Saturation of magnetic circuit is neglected,
- Reaction field caused by ECs is neglected,
- PMs are supposed to be homogeneous and isotropic.

In general, this phenomenon can be described by using integral form for average value of EC losses in PMs:

$$\Delta P_{\mathrm{PM}} = \frac{1}{T}\int_{0}^{T}\int_{R_{\mathrm{r}}}^{R_{\mathrm{r}}+h_{\mathrm{PM}}}\int_{-\xi_{\mathrm{PM}}/2}^{\xi_{\mathrm{PM}}/2}\frac{{J_{\mathrm{PM}}^{\mathrm{Z}}}^{2}}{\gamma_{\mathrm{PM}}}\, l_{\mathrm{PM}} r \mathrm{d}\xi_{\mathrm{r}} \mathrm{d}r\, \mathrm{d}t, \qquad (1)$$

which corresponds to cylindrical system of coordinates (r, ξ_{r}, z) connected to position in rotor (Fig. 1). Conductivity of PMs γ_{PM} and its length l_{PM} are used in this equation. Analytical formulae using infinite series for describing eddy current density are usually used in equation (1). Infinite series and integrals can be relocated by respecting law of commutative. The average value of losses can be then obtained for each of harmonics only by using its RMS value. The proposed method based on harmonic decomposition uses Fourier's coefficients involved in infinite series describing the magnetic field and computes particular losses added by each significant order of harmonics.

III. The Effect of Slotting On Magnetic Flux Density in Permanent Magnets

One of the mathematical tools for describing a magnetic field in the air gap, and permanent magnet respectively, is the complex relative permeance defined in the stator coordinate system as [6], [7]:

$$\bar{\lambda}(r, \xi_\mathrm{s}) = \lambda_\mathrm{a}(r, \xi_\mathrm{s}) + j\lambda_\mathrm{b}(r, \xi_\mathrm{s}). \qquad (2)$$

As long as only radial component of flux density is considered in this method, only the real part of this complex function is used, and the magnetic flux density affected by slotting is then:

$$B_\mathrm{PM}^\mathrm{R}(r, \xi_\mathrm{s}) = B_{\mathrm{PM}_\mathrm{nos}}^\mathrm{R}(r, \xi_\mathrm{s}) \cdot \lambda_\mathrm{a}(r, \xi_\mathrm{s}), \qquad (3)$$

where $B_{\mathrm{PM}_\mathrm{nos}}^\mathrm{R}$ is magnetic flux density without an influence of slotting effect [6].

A. Coordinate System of Stator

The real part of complex relative permeance is defined as follows (corresponding dimensions are depicted in Fig. 1) [3]-[4], [6]:

$$\lambda_\mathrm{a}(r, \xi_\mathrm{s}) = \\ = \delta' \frac{1 + \frac{k_\mathrm{c} - 1}{K_0} \sum_{k=1}^{\infty} [Q_\mathrm{k} R_\mathrm{a}(r, k) \cos(k Q_\mathrm{s} \xi_\mathrm{s})]}{r ln\left(\frac{R_\mathrm{s}}{R_\mathrm{r}}\right) + \frac{k_\mathrm{c} - 1}{K_0} \frac{R_\mathrm{s}}{Q_\mathrm{s}} \sum_{k=1}^{\infty} \left[\frac{(-1)^k}{k} Q_\mathrm{k}\right]} \qquad (4)$$

which consists of:

- effective air gap length

$$\delta' = \delta + \frac{h_\mathrm{PM}}{\mu_\mathrm{rPM}}, \qquad (5)$$

where μ_rPM corresponds to relative permeability of permanent magnets and the real air gap length is defined as:

$$\delta = R_\mathrm{s} - (\mathrm{R_r} + h_\mathrm{PM}), \qquad (6)$$

- Carter's factor

$$k_\mathrm{c} = \frac{t_\mathrm{s}}{t_\mathrm{s} - \gamma \cdot \delta'}, \qquad (7)$$

$$\gamma = \frac{\left(\frac{b_\mathrm{o}}{\delta'}\right)^2}{5 + \frac{b_\mathrm{o}}{\delta'}}, \qquad (8)$$

$$t_\mathrm{s} = \frac{2 \cdot \pi \cdot R_\mathrm{s}}{Q_\mathrm{s}}, \qquad (9)$$

where b_o denotes to width of slot opening, t_s to stator slot pitch and Q_s to number of stator slots,

- function K_0

$$K_0 = \sum_{k=1}^{\infty} Q_\mathrm{k} R_\mathrm{a}(r, k) \cos(k\pi), \qquad (10)$$

- function Q_k

$$Q_\mathrm{k} = \int_0^{\frac{\xi_\mathrm{o}}{2}} \left(\frac{1}{\sqrt[3]{\frac{\xi_\mathrm{o}}{2} - \xi}} - \frac{1}{\sqrt[3]{\frac{\xi_\mathrm{o}}{2} + \xi}} \right) \sin(k Q_s \xi)\, \mathrm{d}\xi, \qquad (11)$$

- function $R_\mathrm{a}(r, k)$

$$R_\mathrm{a}(r, k) = \left(\frac{r}{R_\mathrm{s}}\right)^{kQ_\mathrm{s} - 1} \cdot \frac{1 + \left(\frac{R_\mathrm{r}}{r}\right)^{2kQ_\mathrm{s}}}{1 - \left(\frac{R_\mathrm{r}}{R_\mathrm{s}}\right)^{2kQ_\mathrm{s}}}. \qquad (12)$$

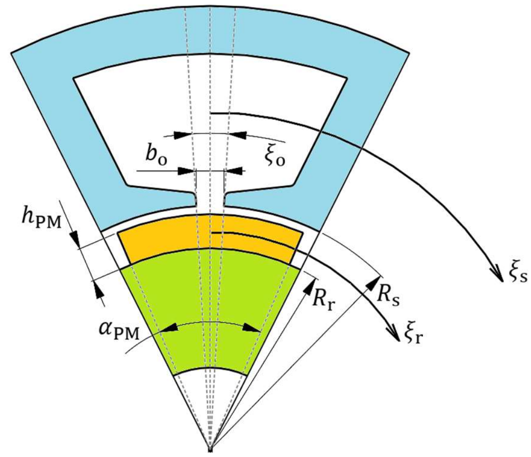

Fig. 1. Cross-section of discussed type of machine (general SPMSM) with reference dimensions used in the algorithm

B. Coordinate System of Rotor

For the sake of being able to evaluate magnetic induction flux in rotating permanent magnet it is necessary to transform relation of equation (4) from stator to rotor coordinate system by defining a rotor angular position ξ_r. This is performed by respecting the following relation:

$$\xi_\mathrm{s} = \xi_\mathrm{r} + \omega_\mathrm{m} t, \qquad (13)$$

where mechanical angular velocity of rotor, and magnet respectively, ω_m and time t are used. This step changes a notation of equation (4) in further text.

It can be seen, still in the equation (4) already updated by (13), that it consists of two parts in the manner of time and angular dependency: stationary $\lambda_\mathrm{a}^\mathrm{s}$ and variable $\lambda_\mathrm{a}^\mathrm{v}$. For evaluation of eddy currents only the time variable part is worth it. Hence the first step of simplification has been obtained. Due to the reason, that this method is based on Fourier's coefficients, the time varying part of the relative permeance equation, consisting of few infinite series, has to be modified by using general rules for operation with such a series:

$$\lambda_\mathrm{a}^\mathrm{v}(r, \xi_\mathrm{r}, t) = \\ = \delta' \frac{(k_\mathrm{c} - 1) \sum_{k=1}^{\infty} [Q_\mathrm{k} R_\mathrm{a}(r, k) \cos(k Q_\mathrm{s}(\xi_r + \omega_\mathrm{m} t))]}{r ln\left(\frac{R_\mathrm{s}}{R_\mathrm{r}}\right) K_0 + (k_\mathrm{c} - 1) \frac{R_\mathrm{s}}{Q_\mathrm{s}} \sum_{k=1}^{\infty} \left[\frac{(-1)^k}{k} Q_\mathrm{k}\right]} \qquad (14)$$

and after that it is transformed to a simple Fourier's series by using a division derived from the Cauchy product:

$$\lambda_\mathrm{a}^\mathrm{v}(r, \xi_\mathrm{r}, t) = \\ = \sum_{k=1}^{\infty} \left[\lambda_{\mathrm{a_k}}^\mathrm{v}(r, k) \cos(k Q_\mathrm{s}(\xi_r + \omega_\mathrm{m} t))\right], \qquad (15)$$

where the k-th harmonic coefficient is defined as:

$$\lambda_{\mathrm{a_k}}^{\mathrm{v}}(r,k) =$$

$$= \delta' \frac{(k_\mathrm{c}-1)Q_\mathrm{k}R_\mathrm{a}(r,k)}{r\ln\left(\frac{R_\mathrm{s}}{R_\mathrm{r}}\right)K_0 + (k_\mathrm{c}-1)\frac{R_\mathrm{s}}{Q_\mathrm{s}}\sum_{k=1}^{\infty}\left[\frac{(-1)^k}{k}Q_\mathrm{k}\right]}. \tag{16}$$

The stationary component of relative permeance $\lambda_\mathrm{a}^\mathrm{s}$ is derived in the same manner as the variable one. It yields the final form of the relative permeance formula:

$$\lambda_\mathrm{a}(r,\xi_\mathrm{r},t) =$$

$$= \sum_{k=1}^{\infty}\left[\lambda_{\mathrm{a_k}}^{\mathrm{s}}(r,k) + \lambda_{\mathrm{a_k}}^{\mathrm{v}}(r,k)\cos\left(kQ_\mathrm{s}(\xi_r+\omega_\mathrm{m}t)\right)\right]. \tag{17}$$

IV. Evaluation of Eddy Current Losses

A. *Distribution of Magnetic Flux Density in PM*

At this point, it is possible to define the magnetic flux density in rotor coordinate system $B_\mathrm{PM}^\mathrm{R}(r,\xi_\mathrm{r},t)$. Because the model focuses only on interior volume of PM, an additional simplification has been established: $B_{\mathrm{PM_{nos}}}^{\mathrm{R}}$ is considered constant over the entire volume of PM.

$$B_\mathrm{PM}^\mathrm{R}(r,\xi_\mathrm{r},t) = B_{\mathrm{PM_{nos}}}^{\mathrm{R}} \cdot \lambda_\mathrm{a}(r,\xi_\mathrm{r},t) \tag{18}$$

Comparison between analytical and FEA results of distribution of magnetic flux density across the magnet's cross-section is depicted in Fig. 2. As a 3D FEA software was used ANSYS Electronics Desktop, version 2021 R1. Investigated SPMSM motor was operated at no-load state and its parameters required for the created algorithm are written in TABLE I. The speed was 9 000 rpm.

TABLE I
PARAMETERS OF INVESTIGATED MOTOR

Inner stator radius	R_s (mm)	35
Outer rotor radius	R_r (mm)	25,8
Axial length of PM	l_PM (mm)	116,2
Stator slot opening	b_o (mm)	9
Permanent magnet thickness	h_PM (mm)	8,1
Permanent magnet angular width	ξ_PM (rad)	1,3
Number of stator slots	Q_s (−)	6
Number of polepairs	p (−)	2
Conductivity of PM	γ_PM (S/m)	555 556
Relative permeability of PM	μ_PMr (−)	1,03
Magnetic flux density	$B_{\mathrm{PM_{nos}}}^{\mathrm{R}}$ (T)	1,08

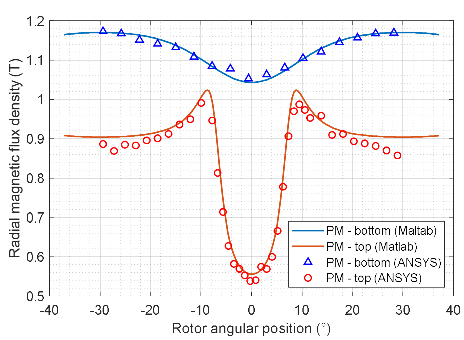

Fig. 2. Magnetic flux density distribution in PM (Matlab x ANSYS)

B. *Distributon of Eddy Current Density in PMs*

For evaluation of EC density the well-known differential form of Faraday's law of induction can be used:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial \mathrm{t}}, \tag{19}$$

where $\vec{E}$ denotes the vector of electric field strength. Due to the previous limitation of magnetic flux density vector to only radial component and eddy current density to the axial one, the equation (19) can be reduced to the form:

$$E_\mathrm{PM}^\mathrm{Z}(r,\xi_\mathrm{r},t) = -\int_{\xi_r} \frac{B_\mathrm{PM}^\mathrm{R}(r,\xi_\mathrm{r},t)}{\partial \mathrm{t}} r\mathrm{d}\xi_\mathrm{r} + C(t), \tag{19}$$

$$J_\mathrm{PM}^\mathrm{Z}(r,\xi_\mathrm{r},t) = \gamma_\mathrm{PM} \cdot E_\mathrm{PM}^\mathrm{Z}(r,\xi_\mathrm{r},t), \tag{20}$$

$$J_\mathrm{PM}^\mathrm{Z}(r,\xi_\mathrm{r},t) = -\gamma_\mathrm{PM}\int_{\xi_r} \frac{B_\mathrm{PM}^\mathrm{R}(r,\xi_\mathrm{r},t)}{\partial \mathrm{t}} r\mathrm{d}\xi_\mathrm{r} + C(t), \tag{21}$$

where constant of integration $C(t)$ obeys to consideration that total current passing through the PM is equal to zero at each time step. However, derivation of this constant consists of surface integration of EC density which involves integration with respect to the radius. This process adds Gauss hypergeometric function 2F1 to definition of $C(t)$. It leads to enormous increase of computational time which contradicts one of the main targets of this project, the time efficiency. The influence on results with and without respecting $C(t)$ was in this specific case of simulation, however, very low. Hence due to the aspiration to offer a fast approach, $C(t)$ is neglected in the algorithm. After that the final formula is:

$$J_\mathrm{PM}^{Z}(r,\xi_\mathrm{r},t) = -\gamma_\mathrm{PM} r\omega_m \cdot \lambda_\mathrm{a}^\mathrm{v}(r,\xi_\mathrm{r},t) \cdot B_{\mathrm{PM_{nos}}}^{\mathrm{R}} \tag{22}$$

Figs. 3 and 4 depict comparison between distribution of eddy current density obtained from the algorithm and FEA.

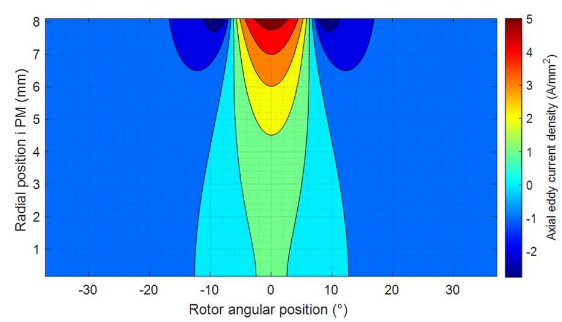

Fig. 3. Distribution of EC density over cross-section of PM (Matlab)

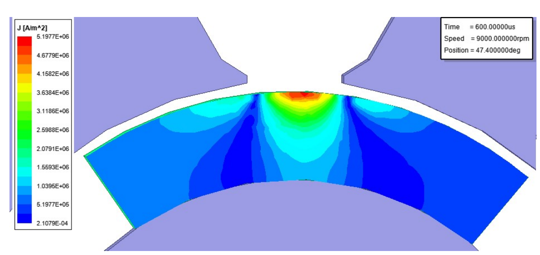

Fig. 4. Distribution of EC density over cross-section of PM (ANSYS)

C. Eddy Current Losses in PM

RMS value of each harmonic component of eddy current density is evaluated as:

$$J_{\mathrm{PM}}^{\mathrm{Z}}(r,k) = \frac{1}{\sqrt{2}}\gamma_{\mathrm{PM}} r\omega_m \cdot \lambda_{\mathrm{a_k}}^{\mathrm{v}}(r,k) \cdot B_{\mathrm{PM\,nos}}^{\mathrm{R}} \quad (23)$$

It can be seen in previous Figs. 2-4 that variation of magnetic flux density and eddy current density is dependent on radius. Hence the average value of EC losses is evaluated for each radius layer and the total value is obtained by numerical integration of these particular results. Losses generated by particular harmonic component in specific radius layer are defined as:

$$\Delta P_{\mathrm{PM}}(r,k) = \frac{J_{\mathrm{PM}}^{\mathrm{Z}}(r,k)^2}{\gamma_{\mathrm{PM}}} l_{\mathrm{PM}} t_{\mathrm{PM}}(r) k_{3D}(r)\mathrm{d}r, \quad (24)$$

where width of PM at a specific radius is used:

$$t_{\mathrm{PM}}(r) = r \cdot \xi_{\mathrm{PM}}. \quad (25)$$

The second new quantity used in equation (24) is correction factor which respects the 3D path of eddy current in permanent magnet and, because the algorithm uses only 2D model, it is appropriate to perform such a correction as is discussed in [8]-[10]. It can be also seen in results of FEA obtained for the investigated motor in Fig. 5.

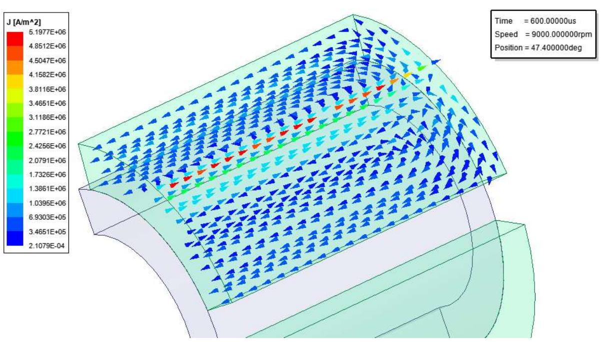

Fig. 5. 3D path of eddy current in permanent magnet (half of length in axial direction of 3D FEA model)

Although the correction factor k_{3D} is taken from [9], it shall not be used in its native form. Each EC loop is situated under one slot opening, slot pitch respectively. Therefore, to provide a correct description of the phenomenon width of the PM is replaced by the slot pitch at the specific radius $t_{\mathrm{s}}(r)$. Hence the correction factor in this algorithm is defined as:

$$k_{\mathrm{3D}}(r) = \left(\frac{l_{\mathrm{PM}}}{l_{\mathrm{PM}} + t_{\mathrm{s}}(r)}\right)^{1{,}7}. \quad (26)$$

The total average value of losses produced in one PM is then obtained from the numerical integration and multiplied by number of permanent magnets $2p$ and number PMs in axial direction n_a if an axial segmentation is applied:

$$\Delta P_{\mathrm{PM}} = n_a \cdot 2p \cdot \int_{R_{\mathrm{r}}}^{R_{\mathrm{r}}+h_{\mathrm{PM}}} \sum_{k=1}^{\infty} [\Delta P_{\mathrm{PM}}(r,k)]\,\mathrm{d}r. \quad (27)$$

D. Verification of the Created Algorithm

The verification has been performed for several different speeds. Comparison between results of total eddy current losses ΔP_{PM} (W) obtained from the 2D created algorithm and 3D FEA is listed in TABLE II.

TABLE II
COMPARISON BETWEEN TOTAL EC LOSSES OBTAINED BY THE CREATED ALGORITHM (MATLAB) AND FEA (ANSYS)

Speed (rpm)	Matlab (W)	ANSYS (W)	Error (%)
3 000	27	32	15,6
6 000	108	127	15
9 000	243	283	14,1
12 000	432	493	12,4

Both, the calculation via the algorithm and FEA verification, were computed at the same PC with 16 cores and 32 GB of RAM. The algorithm needed 4 seconds for execution (including figures' creation). FEA lasted 52 hours.

V. SUBSTITUTION OF INFINITE SERIES

A solution of several infinite series yields the final result. Nevertheless, only a narrow spread of harmonic orders in the whole infinite spectrum performs a relevant influence on the result [11].

The following assumption leads also to the decrease of computational effort. Spatial harmonics produced by the slotting effect can achieve only specific orders, as the following formula determines:

$$\nu = k\frac{Q_{\mathrm{s}}}{p} \ \ldots \ k = \mathbb{Z}. \quad (28)$$

Both of these assumptions are reprojected in Figs. 6 and 7 obtained from the created algorithm. Fig. 6 depicts a spectrum of spatial harmonic components of magnetic flux density in PM including the stationary one. Compared to that Fig. 7 depicts only harmonic orders producing eddy currents in PMs. Both cases prove the validity of equation (28) and especially Fig. 7 depicts how dramatically the significancy of higher spatial harmonics orders decreases.

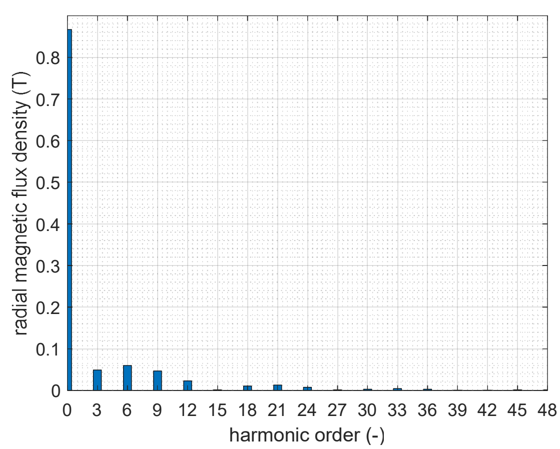

Fig. 6. Spatial harmonics spectrum of radial component of magnetic flux density in permanent magnet

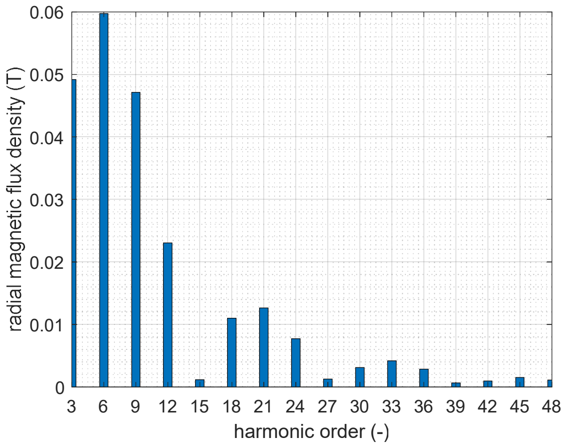

Fig. 7. Spatial harmonics spectrum of radial component of magnetic flux density in permanent magnet (only harmonic orders producing EC)

VI. Conclusion

The created algorithm for evaluating eddy current losses in surface-mounted permanent magnets of synchronous machine caused by slotting effect at no-load state has been discussed in the text.

The main purpose of this research has been to create a fast and relatively precise tool for estimating eddy current losses in PMs during the initial electromagnetic design of SPMSM at no-load state. Hence the algorithm consists of several useful features to fulfil this requirement.

It uses a specific approach for estimation of losses by harmonic decomposition of Fourier's series describing the eddy current density and solving addition of losses by each order of harmonics.

For estimation of the eddy current density, the complex relative permeance function (stated in [6]) is used. It allows to consider the real behavior of magnetic flux density in the interior volume of PMs with respect to the radius.

The problem is reduced to 2D model and effect in vicinity of permanent magnet's axial edges causing a bend of path, which EC are passing through, is respected by using the correction factor (modification of formula from [9]).

Optimized range of spatial harmonics is considered, and evaluation is performed only for significant harmonic orders.

The model has been verified by 3D FEA and results are listed in TABLE II. Time consumption of calculation by the algorithm was 4 seconds in comparison to FEA which lasted 52 hours. Relative error was under 15 % at rated speed. Due to the purpose of the algorithm, to substitute the FEA in the design process of such a machine, and also that significant save of time is the error considered acceptable.

Consideration of the effect of PM eddy currents' reaction field could be an appropriate improvement [5], [12]. It will take a place in the further development of this algorithm. This project is the initial step into analysis of EC losses in PMs, and it is planned to enhance it by implementing the calculation of armature's reaction field as well.

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VIII. Biographies

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