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A generic model of hyperspace curvature preservation for a dynamic radial basis function implicit surface

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1 Introduction

The primary objective of the proposed methodology is to be implemented in the domain of computerized muscle modelling (see, e.g. Cervenka et. al. (2020)). However, preserving shape under different deformation scenarios is crucial across numerous research domains. Consequently, this paper briefly exposes the problem in a broader context. Multiple approaches exist to describe the surface, each with inherent strengths and limitations. In this context, the implicit surface utilized for deformation will be characterized using a collection of radial basis functions (RBFs). Although the choice of RBF may vary depending on the specific domain, this paper employs the widely recognized Gaussian RBF. This selection ensures a smooth and infinitely differentiable surface model, which is advantageous for the intended purpose.

2 Proposed approach

The proposed approach entails the creation of an RBF implicit surface and its subsequent deformation during motion. The primary objective of this approach is to analytically describe the gradient of the most crucial variables associated with each RBF.

2.1 Radial basis functions

The Gaussian radial basis function (RBF) is a mathematical function that incorporates the distance between a vertex, denoted as $\mathbf{x}$, and a predefined centre point, represented as ξ. Each RBF is associated with a shape parameter α, which determines its standard deviation. The entire surface is subsequently characterized by a weighted summation of N individual radial basis functions (RBFs), which can be more effectively represented using the following equation (λ_i is the weight of each RBF):

$$f\left(\mathbf{x}\right) = \sum_{i=1}^{N} \lambda_i e^{-\alpha||\mathbf{x}-\xi||_2^2} = \sum_{i=1}^{N} \lambda_i e^{-\alpha r^2} \tag{1}$$

Determining the right centre points ξ and the shape parameter α is challenging. The optimal values for these parameters can be obtained through various methods, as outlined in e.g. Skala et. al. (2020). These techniques offer guidance and strategies for selecting appropriate centre points and shape parameters, considering data characteristics, problem requirements, and desired outcomes.

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2.2 Dynamic behaviour

The radial basis function needs to be recalculated to restore non-fixed components of a static surface characterized by variables ξ and α. Instead of starting from scratch, an efficient approach is to leverage variables from the previous iteration and make incremental adjustments, eliminating the need for a complete reevaluation of the surface. By incorporating modifications based on previous step, the deformation process can be streamlined for improved efficiency.

2.3 Hyperspace curvature

The central concept of the proposed approach is to maintain the inherent curvature of the surface in 3D hyperspace, even though the surface itself is two-dimensional. To estimate the hyperspace curvature, the mean curvature can be employed. The mean curvature is computed as the average of the eigenvalues of the Hessian matrix. The equation necessitates the calculation of second partial derivatives concerning each combination of coordinate pairs. Fortunately, due to the infinite differentiability property of the Gaussian radial basis function (RBF), the curvature κ in a d-dimensional space can be described analytically as:

$$\kappa_f = \frac{2\alpha}{d} \sum_{i=1}^{N} \lambda_i e^{-\alpha r^2} \left(2\alpha r^2 - d\right) \tag{2}$$

2.4 Centre point estimation

The next step involves estimating the centre points ξi. This process begins by constructing the cost function C_f using the L2 norm between the initial mean curvature $\kappa_{f_{\text{init}}}$ and the current mean curvature at each point in space. These norms are then summed up (integrated) over the entire surface. Subsequently, the gradient of the sum of L2 norms concerning ξ_{ij} (where j represents the individual coordinate of the i-th centre point) provides the direction in which ξ_i should be adjusted. The equation for the gradient in the j-th dimension is given by:

$$\nabla C_{fj} = \frac{8\alpha^2}{d} \int_{\mathbb{R}^d} (\kappa_f - \kappa_{f_{\text{init}}}) \sum_{i=1}^{N} \lambda_i e^{-\alpha r^2} (x_j - \xi_{ij}) \left(2\alpha r^2 - 2 - d\right) d\mathbf{x} \tag{3}$$

3 Conclusion

Indeed, it is possible to analytically describe the displacement of RBF centres during the motion of the implicitly defined surface. The proposed approach includes the analytical representation of this displacement. It is worth mentioning that both the implementation and evaluation of this approach are planned for the near future.

References

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