FACULTY OF APPLIED SCIENCES
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OF WEST BOHEMIA

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Analysing the Properties of Hierarchical RBFs for Interpolation

Alex König[1]

1 Introduction

Interpolation methods using radial basis functions (RBF) are utilized for the creation and reconstruction of elevation and surface models, solving partial differential equations and reconstruction of noisy images. RBF interpolation methods are based on linear combinations of symmetric radial basis functions, each defined by its own center. This task, with a set of distinct points $P = \{p_1, p_2, ..., p_N\}$ with associated values $h = \{h_1, h_2, ..., h_N\}$ as the input, and an interpolating function as the output, can be fulfilled by solving the following system of linear equations:

$$h_i = \sum_{j=1}^{N} \omega_j \phi(||p_i, p_j||), \tag{1}$$

where ϕ is radial basis function, it's parameter $r = ||p_i, p_j||$ is the distance of the point p_i from the center p_j and N is the number of input points and the number of basis functions. The weight ω_j is associated with a specific basis function. These weights have to be determined prior to the computation of the interpolating function, as the solution to the following equation:

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \tag{2}$$

where $\boldsymbol{x}$ is a vector of the weights ω, $\boldsymbol{b}$ is a vector of the values h, and $\boldsymbol{A}$ is a 2D matrix, at the position (i, j) is the value of a basis function ϕ with center in point p_j for point p_i. RBFs are usually used for smaller datasets, due to their high memory and time requirements.

Hierarchical RBF (HRBF) reconstruction is based on the division of the input domain into smaller domains with fewer points, finding local interpolation functions and then joining those local functions together into one global function interpolating the input points. Reconstruction of the local functions takes place in a relatively small isolated part of the input domain, therefore the basis functions need to have a restricted maximal distance of influence. Such basis functions are called *local* or *compactly supported*. This work uses functions introduced by Wendland (1998), defined only for $\alpha r \in$ [0,1]. The distance from the center r is first multiplied by a shape parameter α, $\alpha > 0$, modifying the real reach of a local RBF.

2 Compared methods

This work compares two methods, their main difference is the space subdivision technique. Both were tested on regular and irregular sampling of smooth functions and a real terrain elevation data-set.

First method is balanced binary tree method, as described in Pouderoux et al. (2004), which recursively divides the input domain into two overlapping domains. The second compared method is grid method described in Šmolík and Skala (2017), where the input domain is

[1] student of the master degree program Applied Sciences, field of study Computer graphics, e-mail: akonig@students.zcu.cz

divided into subdomains using a regular grid. Each created cell is then enlarged by a padding. The local interpolation functions are created using local RBF and all points in the cell. As the number of points is small, the weights can be computed as a system of linear equations. When the value of the global interpolation function for a point p is evaluated, the local reconstruction functions of all subdomains containing this point are combined. To compute the global function the partition of unity principle described in Ohtake et al. (2003) is used.

3 Conclusion

For HRBF to approximate sampled functions and surfaces well the input data set should be sampled along the edges of the domain to avoid needlessly big errors on the edges, and densely enough to avoid errors due to insufficient number of points.

Balanced binary tree method offers automatic cell creation, but needs more memory space due to recursion, and may create elongated cells. This can lead to artifacts in the overlaps. Its accuracy depends mainly on the choice of shape parameter and the acceleration depends on the number of points in separate cells, which should be small in comparison to the number of input points.

Grid method offers a simple implementation, but it is required to specify the size of a cell along with the padding, and it does not guarantee points in every cell. In such case, the user needs to change the cell size. Its accuracy is more dependant on the sampling rate of input data and the number of cells. However, better accuracy would be achieved if more sophisticated grid division was implemented.

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