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Effect of pre-stress on modification of modal properties of planar structures

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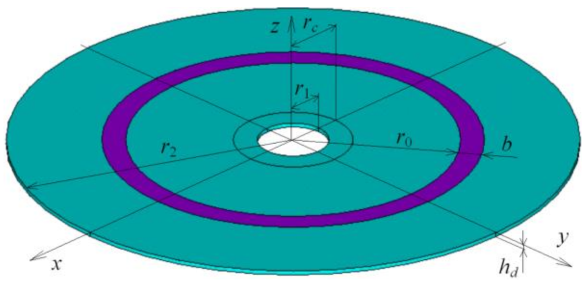
Planar structures such as discs and plates are structural components widely used in structural and manufacturing applications. In the case of dynamic loading of a given planar structure, it is important that structure has the required modal properties. To obtain suitable modal properties of planar structures, there are several methods, which are usually based on the application of additional structural elements, attached to the basic planar structure. By using such structural modifications of the original planar structure, a redistribution of mass and stiffness properties and also the change of its modal properties is achieved. In order to achieve the required modal properties of planar structures, a method based on the creation of pre-stress in the plane of the structure is presented in this article.

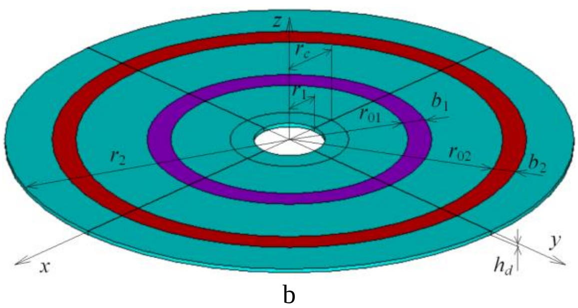
A specific case of planar structures such as a circular disk are saw blades, which are widely used for wood cutting and shaping. In the cutting process, there are occasional large transverse displacements of the circular saw blade, which are caused by resonant states. To improve dynamic stability, i.e. elimination of resonance states by modifying its modal properties, it is possible to use the creation of pre-stressed areas in the plane of the circular saw blade. The influence of the parameters of the pre-stressed areas created in the circular saw blade on the modification of its modal properties is investigated and analyzed in this paper.

One of the methods for the creating of stress in-plane of circular saw blade is rolling a certain area of the blade surface. In the rolling process, the disc is compressed in rolled zone between two opposing rollers. During such rolling, a permanent plastic deformation is created in the rolled part of circular saw blade. As a result of this permanent deformation, the compressed material creates residual stress in the plane of the circular saw blade.

The values of natural frequencies for one and two plastic zones (Fig.1), their various rolling positions and for various rolling depth of the annulus are obtained by modal analysis using finite element method (FEM). The role of residual stresses obtained by rolling is assessed from the change in natural frequencies and modal shapes.



a



b

Fig. 1. The circular disc with plastically deformed zones: a - one plastic zone; b - two plastic zones

We consider circular saw blade of outer radius r_2, inner radius r_1 and thickness h_d (Fig.1). The radius r_c specify where the saw blade is clamped by flanges. For case with one plastically deformed zone (Fig.1a) are dimensions of determined by inner radius r_0 and width b and for two plastically deformed zones (Fig.1b), the dimensions of annulus are r_{01}, b_1 and r_{02}, b_2.

In a structure in which residual stress and strain exist, the stress-strain state can be generally expressed by the relation

$$\boldsymbol{\sigma} = \mathbf{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) + \boldsymbol{\sigma}_0 , \tag{1}$$

where $\boldsymbol{\sigma}$ - stress vector, $\boldsymbol{\varepsilon}$ - strain vector, $\boldsymbol{\sigma}_0$ - residual stress vector, $\boldsymbol{\varepsilon}_0$ - residual strain vector and $\mathbf{D}$ - elasticity matrix.

The FE formulation of equation of motion for a free vibration of circular saw blade without existence residual stresses is expressed by

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{2}$$

and eigenvalue problem has the form

$$(\mathbf{K} - \omega_i^2 \mathbf{M})\boldsymbol{\psi}_i = \mathbf{0} ,$$

where $\mathbf{M}$ - mass matrix, $\mathbf{K}$ - stiffness matrix, $\mathbf{q}$ - vector of nodal displacements, $\ddot{\mathbf{q}}$ - vectors of nodal accelerations, ω_i - i^{th} natural angular frequency, $\boldsymbol{\psi}_i$ - i^{th} mode shape of circular saw blade.

By creating plastic zones in the circular saw blade body, a certain form of structural modification occurs, which leads to a change in the distribution of the spatial properties (mass, stiffness) of the saw blade. It is evident, that the mass distribution of circular saw blade after rolling is not changed, but the bending stiffness can be considerably changed. The finite element equation of motion for a free vibration of circular saw blade with existence residual stresses represented by equation (1) is generally expressed in the shape

$$\mathbf{M}\ddot{\mathbf{q}}_m + (\mathbf{K} + \Delta\mathbf{K}_\sigma)\mathbf{q}_m = \mathbf{0} \tag{3}$$

and then eigenvalue problem has the form

$$(\mathbf{K} + \Delta\mathbf{K}_\sigma - \omega_{m,i}^2 \mathbf{M})\boldsymbol{\psi}_{m,i} = \mathbf{0} , \tag{4}$$

where parameters taking into account the stress distribution in the body of circular saw blade induced by rolling $\Delta\mathbf{K}_\sigma$ - modifying stiffness matrix, $\ddot{\mathbf{q}}_m$ - modified vectors of nodal accelerations, $\mathbf{q}_m$ - modified vector of nodal displacements, $\omega_{m,i}$ - i^{th} modified natural angular frequency, $\boldsymbol{\psi}_{m,i}$ - i^{th} mode shape of circular saw blade.

To determine the change in the stiffness of the disk after rolling and to determine the distribution of residual stresses in the plane of the circular saw blade, the thermal stress method is used. The thermal expansion embedded to annular zones induces the stress distribution in circular saw blade, which is analogous to the stress distribution initiated by rolling. The dependence between the temperature and the indentation depth of the rolled surface can be approximately described by the equation

$$\Delta T \approx \frac{\mu}{\alpha h_d} \Delta z , \tag{5}$$

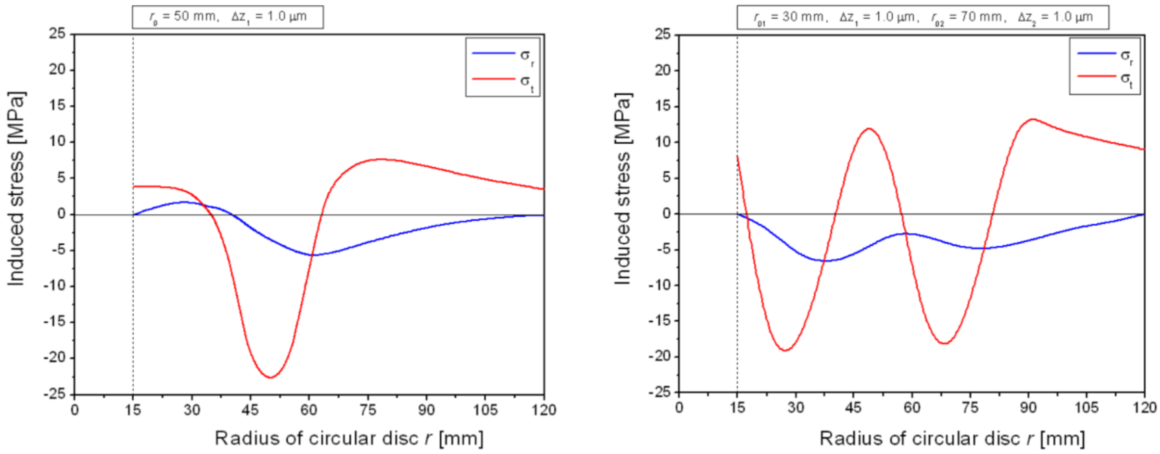
where μ - Poisson´s number, α - temperature expansion coefficient, h - thickness of circular saw blade and Δz - indentation depth of the rolled surface.

The geometrical dimensions and material properties considered circular saw blade (Fig.1) are given in Table 1.

Table 1. The dimensions and material properties of circular saw blade

r_1 [mm]	r_2 [mm]	r_c [mm]	h_d [mm]	$b = b_1 = b_2$ [mm]	E [GPa]	ρ [kg/m^3]	μ [-]
15	120	25	1.8	10	210	7800	0.3

The distributions of radial and tangential residual stresses in the plane of the circular saw blade are shown in Fig.2, i.e. with one plasticized annular zone (Fig.2a) and with two plasticized annular zones (Fig.2b).



a b

Fig. 2. Distribution of radial σ_r and tangential σ_t residual stresses initiated in disc plane: a - one plasticized zone; b - two plasticized zones

The values of natural frequencies of the modal shapes 0/1, 0/0, 0/2, 0/3 (nodal circles/nodal lines) depending on the rolling depth and the position of the plasticized annular zone for a circular saw blade (Fig.1a) are shown in Fig.3. Width of plasticized zone is b = 10 mm.

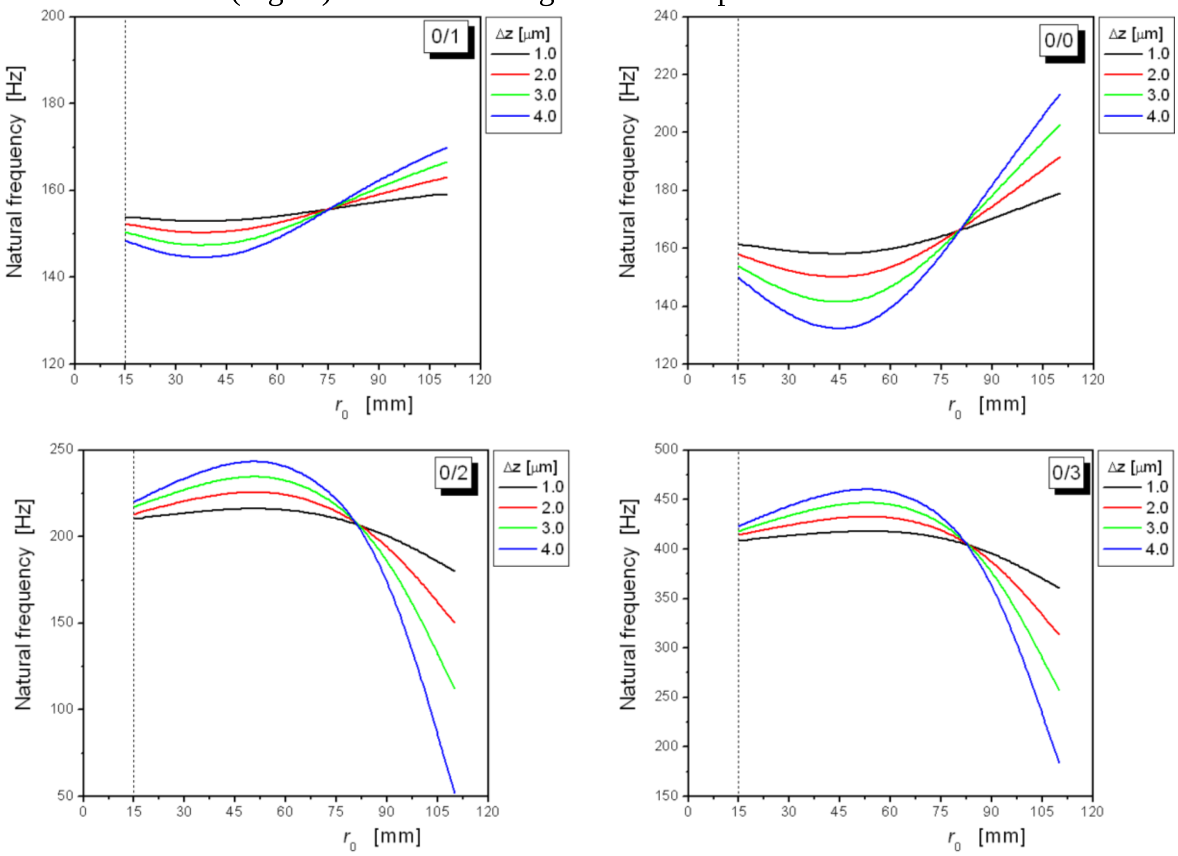


Fig. 3. Dependency of natural frequencies on position of plasticized zone for the first four modal shapes (circular saw blade with one plasticized annular zone)

The tendency of frequency curves for mode shapes 0/1 and 0/0 differs from tendency of frequency curves for mode shapes 0/2 and 0/3. The natural frequencies of the modal shapes 0/2 and 0/3 are increasing with r_0 until the maximum values near $r_0 \cong 55$ mm are reached; then they are decreasing. Contrary to this, the natural frequencies of the modal shapes 0/1 and 0/0 are decreasing with r_0 and for $r_0 \cong 46$ mm reach the minimum and then they increase.

The results for values of natural frequencies of circular saw blade with two plasticized annular zones (r_{01} has constant value 30 mm, radius r_{02} varies from 30 mm to 120 mm and width of both plasticized zones is $b_1 = b_2 = 10$ mm) are shown on Fig 4.

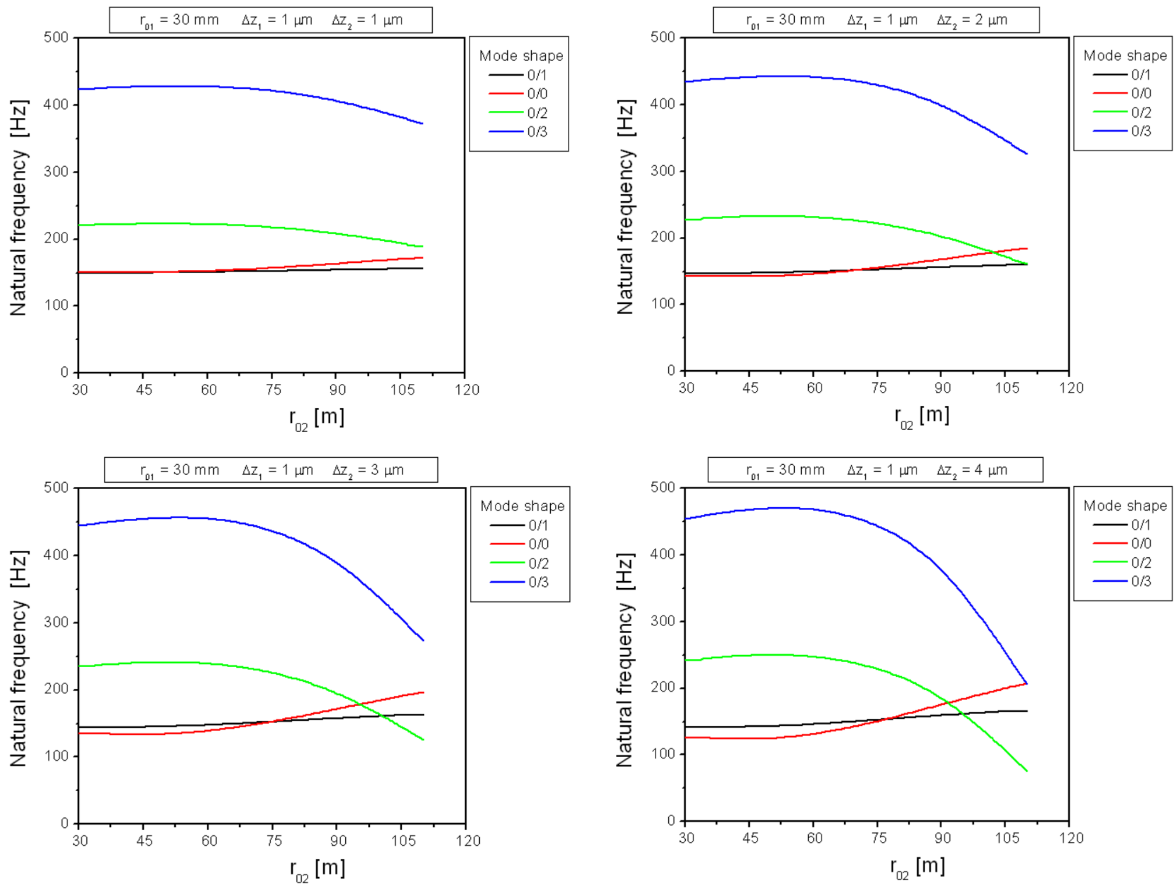


Fig. 4. Dependency of natural frequencies of the first four modal shapes on position of plasticized zone (circular saw blade with two plasticized annular zones)

The effect of two plasticized annular zones on modal properties of circular saw blade is similar as for case with one plasticized zone.

Based on the obtained results, it can be concluded that this method of structural modification can be used to solve the dynamic stability of mechanical systems and is very effective, for example, in modifying the modal properties of circular saw blades.

Acknowledgements

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