COMPUTATIONAL MECHANICS 2023
38th conference with international participation
Srní
October 23 - 25, 2023

Optimizing car tailgate design through truss topology optimization

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1. Introduction

This study addresses the role of truss topology optimization in improving car tailgate designs within the automotive industry. Initially, the principles of truss topology optimization are briefly reviewed. The study then presents a completed analysis focusing on truss topology optimization to develop a more efficient tailgate structure. This includes defining the problem by outlining the solution domain, applied loads, and boundary conditions. Future work aims to integrate this optimized truss design with a parametric design approach that is adaptable to sheet metal forming processes. The paper concludes by discussing the preliminary results and their potential implications for enhancing tailgate reinforcements, as well as outlining the intended methodology for the next phase of the research.

Topology optimization (TO) stands as an invaluable tool for contemporary engineering, focusing on achieving the best or nearly optimal distribution of materials within a specified design area. Over recent decades, there's been a surge of interest in this domain, largely driven by the initiative to cut down the weight of structural elements, while at the same time improving their structural performance. Using TO can potentially lead to savings in terms of material, fuel, manufacturing and other related costs. Historically, TO has been synonymous with elaborate designs that are predominantly feasible through additive manufacturing (AM). While AM remains the most widely used manufacturing technique of results received by TO, it is possible to incorporate TO into the design process of components, which are to be manufactured by traditional approaches [1, 2].

In this research, our objective centres on optimizing the positioning of reinforcements within a car's tailgate. To do so, we employ truss topology optimization (TTO) to identify the candidate areas of potential need for reinforcement. An introductory overview of TTO is provided, followed by a methodology to tackle a specific problem formulation. Next, we shift to modelling and pre-processing, where we describe relevant parts of the modelling environment of our custom TTO software developed in Python, the design space, and we conclude this section with a simple optimization setup with preset boundary conditions, loads and other inputs. Finally, we wrap up this study with a discussion of our preliminary results and we draw up how we intend to use the results of the TTO for the design of the reinforcements.

2. Truss topology optimization

TTO is a relatively mature technology, with its origins in the early 20$^{\text{th}}$ century. It would take until late 1960's, however, when first problem formulations and algorithms started to emerge.

The most widely used approach to a TTO problem (and a TO problem for that matter), is the so-called "Ground structure approach", where the design space is discretized by a structure, within which one seeks the optimal substructure with respect to loads, boundary conditions, etc. [1, 2].

There are many different possible TTO problem formulations, each with its own strengths and weaknesses. In this study, we use the formulation of compliance minimization, constrained by maximum volume to construct the structure and by fulfilling the equilibrium equations. The design variables are the cross-sectional areas of the individual bars. In our case, compliance takes the form of the work of external forces, that is $\delta = \mathbf{f}^T\mathbf{u}$, where δ is the work of external forces, $\mathbf{f}$ is the external force vector and $\mathbf{u}$ is the displacement vector. For linear-elastic material, this cost function can be substituted by the complementary strain energy, which can be written for the truss case as $c = \sum_{e=1}^{n} \frac{N_e^2 L_e}{2EA_e}$. The inclusion of the maximum volume constraint yields

$$\begin{aligned} \arg\min_{\mathbf{A}\in\mathcal{A}} c(\hat{\mathbf{N}}) &= \sum_{e=1}^{n} \frac{N_e^2 L_e}{2EA_e}\,, \\ \mathcal{A} &= \{\mathbf{A} | \sum_e A_e L_e \leq V\}\,, \end{aligned} \tag{1}$$

where N_e is the inner force inside the bar e, similarly L_e denotes the bar's length, E is Young's modulus, A_e is the cross-sectional area, V is the pre-set volume, and finally $\mathcal{A}$ denotes the set from which design variables can be drawn. To fulfil the conditions of equilibrium, we use the principle of minima of the complementary energy. This, coupled with (1), gives the final formulation

$$\begin{aligned} \arg\min_{\mathbf{N}\in\mathcal{N}} \min_{\mathbf{A}\in\mathcal{A}} c(\mathbf{N}) &= \sum_{e=1}^{n} \frac{N_e^2 L_e}{2EA_e}\,, \\ \mathcal{A} &= \{\mathbf{A} | \sum_e A_e L_e \leq V\}\,, \\ \mathcal{N} &= \{\mathbf{N} | \mathbf{B}\mathbf{N} - \mathbf{P} = 0\}\,, \end{aligned} \tag{2}$$

where $\mathcal{N}$ denotes the set of all statically admissible solutions. The optimization problem as defined by equation 2 can be solved, e.g. by the Lagrange multipliers method. The Lagrangian takes the form of

$$\mathcal{L}(\mathbf{A}, \mathbf{N}, \lambda, \mu) = c(\mathbf{N}) + \lambda^T(\mathbf{B}\mathbf{N} - \mathbf{P}) + \mu(\mathbf{A}^T\mathbf{L} - V)\,, \tag{3}$$

which after deriving with respect to the variables and setting as equal to zero, gives us the necessary conditions of optimality. From these, we can derive the following optimization loop:

1. Set $A^{\{0\}}$ as arbitrary positive vector.
2. Compute displacements as $\mathbf{u}^{\{\mathbf{k}\}} = \mathbf{K}^{-1}(A^{\{k\}})\mathbf{f}$.
3. Using the displacements, compute inner forces $N^{\{k\}}$ and find cross-sectional areas for next iteration as $A_e^{\{k+1\}} = \frac{V N_e^{\{k\}}}{\sum_{p=1}^{M} N_p L_p} \quad (l = 1, 2, ..., L)$.
4. Repeat steps 2–3 until a specified convergence criterion is met [3].

3. Modelling and preliminary results

The described TTO algorithm was developed in a custom Python code, which is to be used for the optimized placement of stiffeners of the car's tailgate. The model of the tailgate, which also acts as the design space can be seen in Fig. 1 (left). A mesh was generated in Abaqus, where the

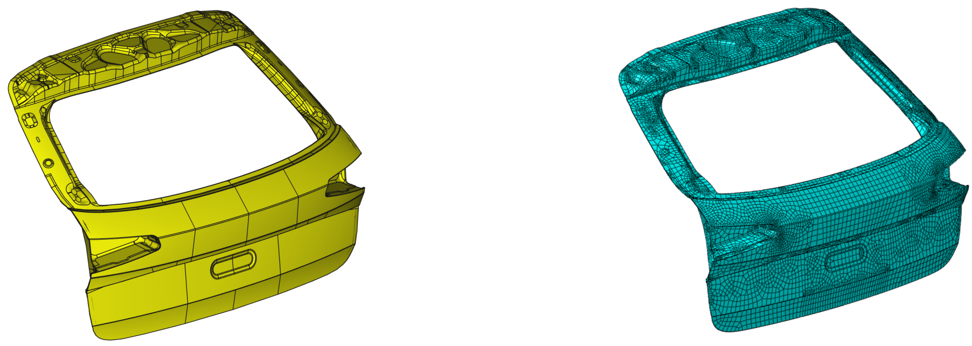

Fig. 1. The car's tailgate model (left); the mesh created by Abaqus (right)

nodes of the mesh were then exported and used as the nodes of the truss structure in the TTO model. The mesh is depicted in Fig. 1 (right).

The ground structure was instantiated by interconnecting all nodes in a small neighbourhood, which was set as 30 mm in this specific case. The approximate dimensions of the tailgate model are 1 300 mm in width and 800 mm in height. This procedure yields the ground structure as shown in Fig. 2 (left). On the right of the same figure is the depiction of the boundary condition of a simple test problem. The nodes that are red have all three degrees of freedom fixed and the green nodes have each a force of 100 N in the vertical direction acting on them. The initial cross-sectional area of all bars was set as 10 mm^2, and the desired volume fraction was set as 10 %.

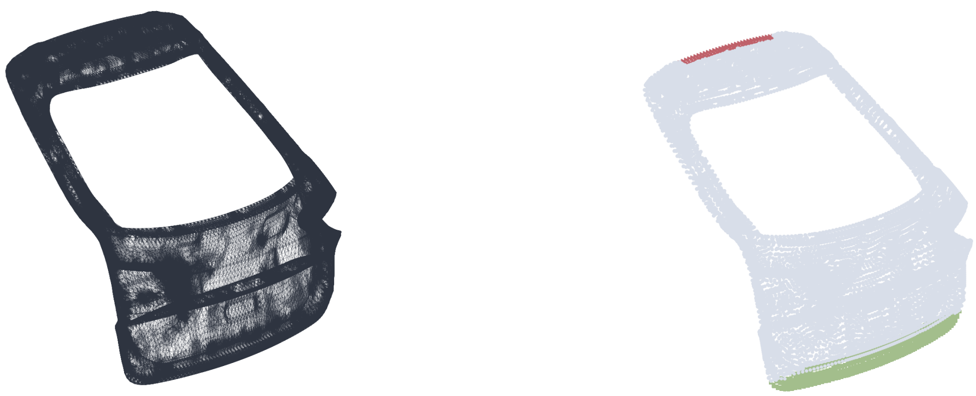

Fig. 2. The ground structure (left) and the boundary conditions (right). The clamped boundary conditions are the red nodes, the nodes with external forces are green

The first optimization result can be see in Fig. 3 (left). The sizes of few bars greatly skew the results and in turn the graphical representation, so the subsequent runs were performed with a maximum cross-sectional area constraint. The result with the maximum area constrained at 50 mm^2 is shown in Fig. 3 (right).

Both of the runs took approximately one hour to complete, which is mainly driven by the suboptimal ground structure. It can be expected that with a better mesh and interconnectivity of the ground structure, fewer bars could be used to represent the problem without a significant loss of accuracy, thus leading to significant performance boost. In terms of programming, we were also forced to use sparse matrix representation of the stiffness matrix, further reducing the speed of the optimization.

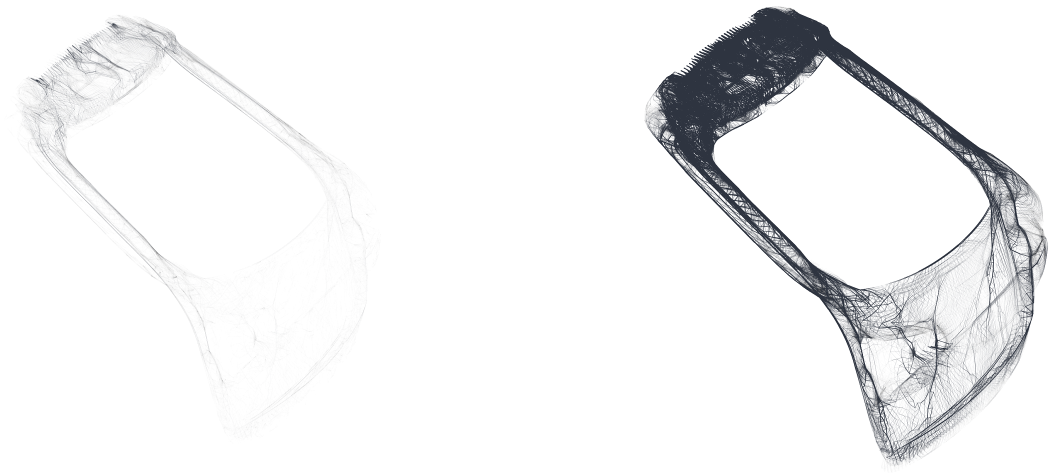

Fig. 3. The result with unconstrained maximum area (left) and the same run with maximum area constrained at $50\,\mathrm{mm}^2$ (right)

4. Discussion and further development

In this work, we have briefly described the concept of topology optimization with the focus on trusses in particular. Next, we have shown one possible optimization formulation of compliance minimization with a volume constraint. Then, we have derived an optimization loop, solving the presented optimization formulation, leveraging the Lagrange's multiplier technique. After that, we moved onto presenting the particular problem we were to solve, consisting of optimized stiffener placement in a car's tailgate. We used our custom program, which deploys the presented optimization loop, to solve a simple static test problem.

The preliminary results of our optimization runs show great potential in terms of identification of optimized stiffener placement in a car's tailgate. Next step is the interpretation of the TTO results. Our idea is to map the TTO results on the boundary surfaces of the design space. Then, design stiffeners for the spots where bar concentration is higher than a given threshold. The stiffeners would then be fine-tuned in a parametric FEM model. All of this is subject to future work.

Acknowledgements

This study received funding from NCK2-Bovenac 4-WP10-003 and the Grant Agency of the Czech Technical University in Prague, under grant No. SGS21/151/OHK2/3T/12.

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