COMPUTATIONAL MECHANICS 2023
38th conference with international participation
Srní
October 23 - 25, 2023

Analysis of autoparametric double pendulum

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Rott's pendulum [3] is a special case of a double pendulum that possibly exhibits autoparametric resonances. As shown in Fig. 1, the basic lumped-mass model consists of two subsystems – the left one is mass m located at the end of the massless angled arm that is pivoted in P_1 and the right one is a simple mathematical pendulum with mass m attached to the left arm in pivot P_2. If pivots P_1 and P_2 are aligned horizontally in the rest position, it yields strong quadratic coupling between both pendula. If, moreover, the eigenfrequencies of both pendula are tuned in the particular integer ratio, most often 1:2, internal resonance with a slow energy exchange between both subsystems occurs.

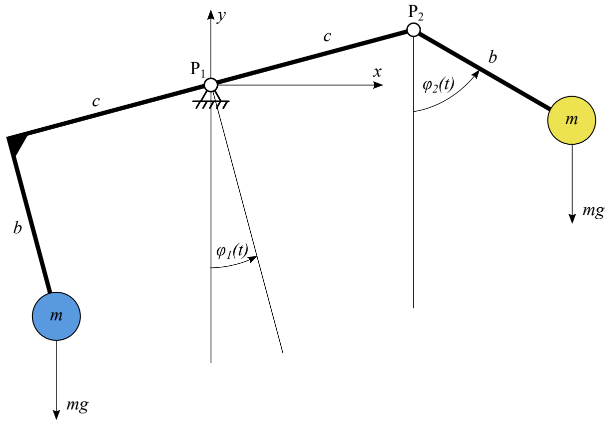

Fig. 1. Scheme of the lumped-mass Rott's pendulum

Denoting the angular position of both pendula φ_1, φ_2 and geometric parameters b, c as depicted in Fig. 1, the system in the nondimensional form can be formulated in the following form:

$$\begin{bmatrix} 1+2\alpha^2 & -\alpha\sin(\varphi_1-\varphi_2) \\ -\alpha\sin(\varphi_1-\varphi_2) & 1 \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \end{bmatrix} + \begin{bmatrix} \sin\varphi_1 \\ \sin\varphi_2 \end{bmatrix} + \begin{bmatrix} \alpha\cos(\varphi_1-\varphi_2)\dot{\varphi}_2^2 \\ -\alpha\cos(\varphi_1-\varphi_2)\dot{\varphi}_1^2 \end{bmatrix} = \mathbf{0}, \tag{1}$$

where the geometric tuning parameter $\alpha = c/b$ and temporal derivatives are expressed with respect to nondimensional time $\tau = \Omega t$ for $\Omega = \sqrt{\frac{g}{b}}$. Initial states are given by nonzero initial positions of both pendula [2].

For the analysis, the energy-based method introduced in [4, 5] was used. This method divides the total energy of the system to term corresponding to the energy of the simple (here pendular) motions of subsystems, and a coupling energy. In the studied case, we formulated following expression for total energy (Hamiltonian)

$$E = E_1 + E_2 + E_C, \tag{2}$$

where E_i are energies of both pendula $i = 1, 2$ (including both kinetic and potential energy of the pendula) and a coupling energy E_C. It is shown that the states with slow energy exchange are characterised by a minimum coupling energy. This is used for in-depth analysis of such a Hamiltonian system: a set of internal resonances $1 : \kappa,\ \kappa = 2, 3, 4, 5$ is revealed with respect to initial energy E, see Fig. 2.

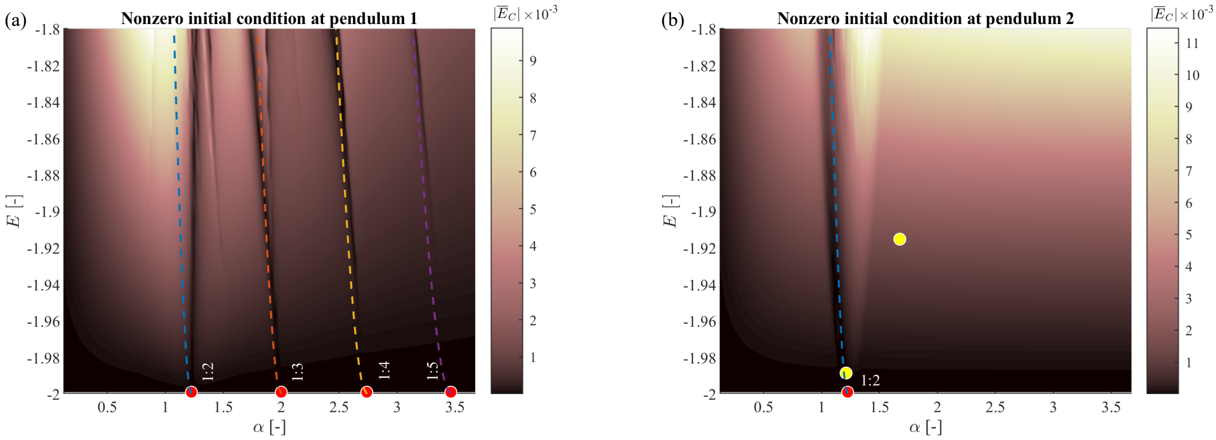

Fig. 2. Temporal averages of the coupling energy $\overline{E}_C = \overline{E}_C(\alpha, E)$ as a function of the geometric tuning parameter α and initial energy E. Dashed lines correspond to analytically obtained internal resonances $1 : \kappa$, $\kappa = 2, 3, 4, 5$

Moreover, we have shown how the coupling energy behaves for higher energy levels characterised by the chaotic and spinning motion of subsystems. To reveal chaotic regimes, fast Lyapunov indicators (FLI) are used as suggested in [1].

The energy-based method appears to be a robust method for the analysis of autoparametric systems. The great advantage is its applicability for all the motion regimes (regular periodic, quasiperiodic, chaotic motion) and is not limited to small amplitudes or a usage of the lower-order approximations of motion equations.

Acknowledgement

This research work was supported by the Czech Science Foundation project 23-07280S entitled ”Identification and compensation of imperfections and friction effects in joints of mechatronic systems”.

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