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Adaptation of methods for cyclo-stationary processes for noisy structural health data

C. Fischer[a], J. Bayer[a], J. Náprstek[a], S. Urushadze[a]

[a] Institute of Theoretical and Applied Mechanics of the Czech Academy of Sciences, Prosecká 76, 190 00 Prague, Czech Republic

1. Introduction

In structural health analysis, various techniques, including indirect measurement via monitoring vehicles, often yield data with significant randomness and insufficient frequency separation. Conversely, the desired attributes under scrutiny are periodic in nature. Thus, methodologies designed to identify cyclo-stationary properties within noisy data can be adapted for such scenarios, assuming an adequate length of the recorded data.

The concept of determining a bridge's natural frequency by utilizing a passing vehicle has been a pursuit since the publication of a mathematical solution for a suspended mass on a beam, [6]. For a comprehensive overview of progress in this area, consider referring to a recent review of the Vehicle Scanning Method (VSM), previously known as 'drive-by identification' [4].

However, it is essential to recognize that the idealized scenario presented in the closed-form solution, upon which VSM is built, is often impractical in real-world bridge situations. Several challenges arise, including the fact that the suspended mass cannot directly roll on the road or rail—it typically needs to be either pulled by another vehicle or equipped with its own drive. Furthermore, most bridges cannot be accurately represented as simple supported beams. Additionally, damping effects cannot always be disregarded, and both the vehicle and the bridge exhibit non-zero vibrations as the scanning vehicle enters the bridge.

These limitations prompted the development of finite element (FE) pre-processing-based solutions. These solutions involve the use of specialized elements to simulate the interaction between the vehicle and the bridge. They employ the Vehicle-Bridge Interaction (VBI) element [7], the MINE element [5] or FE software programs implementing kinematic formulations, such as LS-Dyna. These FE-based approaches aimed at creating models that reliably replicate the simplified experimental setups in order to compare numerical results and measured data [1].

The experimental bridge model corresponds to the outline in Fig. 1. It is made of a steel U-profile $0.21 \times 0.05 \times 0.004$ m with $L = 3.98$ m and a total mass of 33.3 kg. These values imply two first natural frequencies 6.99 and 27.63 Hz. Other structural parameters include viscous proportional damping ($\alpha = 0.2$, $\beta = 2.5 \times 10^{-5}$) and damping ratio of $c_s = 0.01$ for the spring dash pot, the spring mass $m_s = 245$ g and the vehicle mass $m_v = 631$ g.

In the experimental case, the measured data display a pronounced random component, which effectively obscures the frequency characteristics of the idealized experimental model. The frequency content of the signal measured on the passing vehicle is highly influenced by the vehicle's velocity and boundary conditions, and to some extent, also by the vehicle's position on the beam. Nevertheless, for sufficiently long beams, it is reasonable to assume that the boundary and positional effects can be considered negligible.

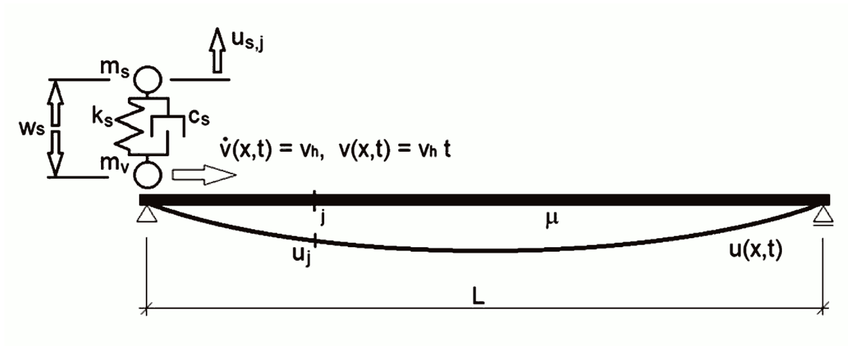

Fig. 1. Schema of the test beam and a moving sprung mass

Despite making such an assumption, the response is non-stationary and displays periodically repeating characteristics, as illustrated by the example plot of the measured response in Fig. 2. In general, this process lacks both stationarity and ergodicity. Consequently, the application of procedures commonly employed for evaluating stochastic parameters over time, as is typical in the context of stationary ergodic processes, is effectively precluded.

2. Cyclo-stationary and cyclo-ergodic processes

The periodic nature of the stochastic moments suggests the application of Cyclo-Stationary Process (CSP) theory. This approach describes situations in which consecutive quasi-periods resemble those observed in synchronously running two or more parallel realizations of the process. CSPs belong to a subclass of general non-stationary processes with periodically repeating characteristics. At the same time, it can be shown that the properties mentioned above allow non-ergodic CSPs to be understood as a 'Cyclo-Ergodic Process' (CEP), as illustrated in Fig. 3. Following the original definition of stochastic moments, they should be evaluated across individual realizations of the process. For example, by summing relevant values of $u^{(i)}(t)$, where the superscript (i) represents the realization number (e.g., black, red, blue in Fig. 3, corresponding to quasi-periods along the i axis). However, in the context of CEP, the red or blue periods along the t axis hold an equivalent stochastic value to those along the (i) axis.

The foundational works providing a theoretical background for CSP are primarily attributed to Gardner, see, e.g., [2] or recent monograph [3]. CSPs that exhibit cyclo-stationarity in second-order statistics, such as the autocorrelation function, are referred to as wide-sense CSP and are analogous to wide-sense stationary processes.

One of the key parameters characterizing a measured CSP is the length of one quasi-period (QPL). Typically, this value is not known in advance, but any a priori knowledge gained from analytical or numerical analysis can significantly enhance the identification process. When we consider the measured data as a quasi-periodic random process, the QPL becomes a stochastic

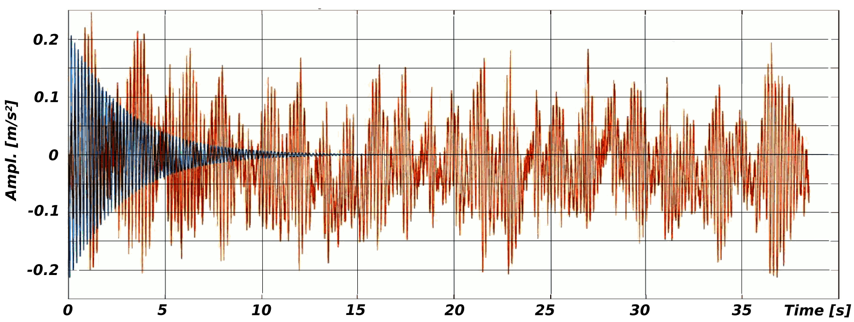

Fig. 2. Measured (red) and calculated (blue) response of the sprung mass

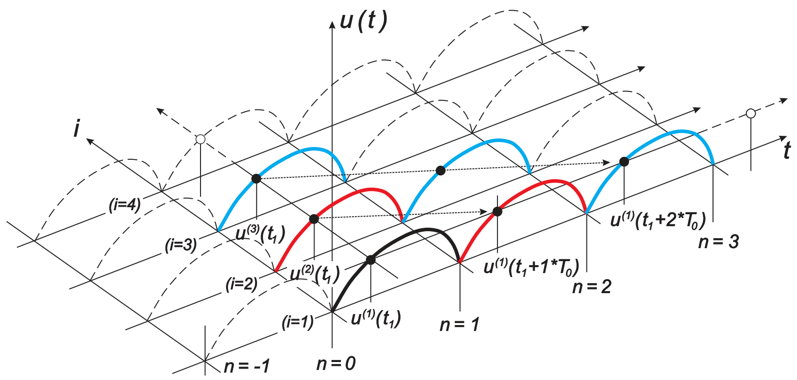

Fig. 3. Cyclo-Stationary and Cyclo-Ergodic Process; superscript in $u^{(i)}(t)$ means number of the process realization; n is number of the subsequent period of the length T_0

variable, as depicted in Fig. 4. The spread or variance of the QPL depends on various factors and tends to be sensitive on the presence of non-linear effects in the model. Consequently, these processes are referred to as Almost Cyclo-Stationary Processes (ACSP). It is evident that the variability of the QPL (within the context of a single parameter setting) should be taken into consideration. Nonetheless, it is worth noting that the variance of this statistic is typically small, allowing for a reasonable approximation of QPL variability with a suitable value.

Let each of the N quasi-periods is rescaled to the same number of steps $\delta_n = T_{0n}/N_s$. Here, δ_n represents the time increment within the n-th quasi-period, where T_{0n} is the length of the quasi-period, and N_s is a constant representing the number of steps within each quasi-period, cf. Fig. 4. This assumption allows δ_n and T_{0n} to be universally denoted as δ and T_0.

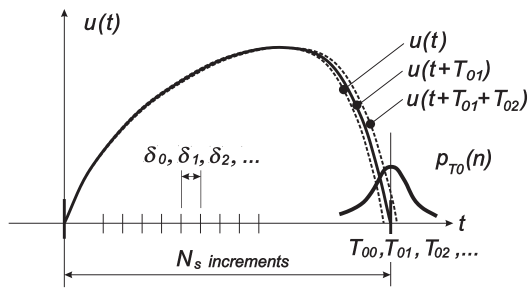

Fig. 4. TCSP – sampling with respect to QPL variability

The M-order cyclo-stationarity signifies periodically time-varying stochastic moments up to order M. In accordance with the aforementioned sampling style, primary stochastic characteristics can be defined under the CEP assumption, based on two-point statistics, as follows:

- *Mathematical mean value*:

$$m_u(t_1) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} u(t_1 + nT_0) = m_u(t_1 + T_0)\,. \tag{1}$$

- *Auto-correlation function*:

$$R_u(t_1, t_2) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} u(t_1 + nT_0 + \frac{1}{2}t_2) \cdot u(t_1 + nT_0 - \frac{1}{2}t_2) = R_u(t_1 + T_0, t_2)\,. \tag{2}$$

- *Cross-moment* of two processes $u(t), v(t)$ of the $(M = r + s)$-th order:

$$C_{uv}^{rs}(t_1, t_2) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} u^r(t_1 + nT_0 + \frac{1}{2}t_2) \cdot v^s(t_1 + nT_0 - \frac{1}{2}t_2) = C_{uv}^{rs}(t_1 + T_0, t_2)\,, \tag{3}$$

where $r + s = M,\ t_1, t_2 \in (0, T_0)$.

In (1)–(3), both coordinates t_1, t_2 are considered within one quasi-period $(0, T_0)$. Processes $u(t), v(t)$ in (2), (3) are assumed to be centred.

The shape of the Probability Density Function (PDF) $p(u, t)$ of the $u(t)$ process depends on time $t \in (0, T_0)$. It can be deduced from a relative density of occurrence of a mean value $u_s(t)$ within a sample small interval $\Delta u(t) = u_{max} - u_{min}$ for every point $t \in (0, T_0)$

$$p(u,t) = \lim_{\Delta u \to 0} \lim_{N \to \infty} \frac{n_u(t)}{2N+1}, \quad \text{and} \quad p(u,t_1,t_2) = \lim_{\Delta u \to 0} \lim_{N \to \infty} \frac{n_u(t_1,t_2)}{2N+1}, \tag{4}$$

where $n_u(t)$ is the number of occurrences of a particular value $u(t)$ within the interval $\Delta u(t)$ at the instant t during all $2N$ quasi-periods. Similarly, $n_u(t_1, t_2)$ is the number of occurrences of a particular value of $u(t)$ at moment t_1 and another value of the process $u(t)$ at moment t_2, considering all $2N$ quasi-periods. It should be noted that the order of limits in (4) is not interchangeable.

The stochastic moments of the cyclo-stationary process can be evaluated either using the formulas in (1)–(3), or by employing their definitions with the PDF as described in (4)

$$m_u(t_1) = \int_{-\infty}^{\infty} u(t_1) p(u,t_1) \, \mathrm{d}u, \quad \text{or} \quad R_u(t_1,t_2) = \iint_{-\infty}^{\infty} u(t_1) u(t_2) p(u,t_1,t_2) \, \mathrm{d}u(t_1) \mathrm{d}u(t_2) \,. \tag{5}$$

3. Conclusions

The theory of cyclo-stationary and cyclo-ergodic processes, along with their corresponding procedures, enables users to enhance frequency identification from data acquired through the vehicle scanning method. The assumption of cyclo-stationarity implies a restricted variability of system parameters within the recorded data, which could pose challenges in certain setups. It seems that, e.g., the wavelet-based approach may offer advantages in such situations. This topic warrants further investigation.

Acknowledgements

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