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Modeling and implementation of joints in 2-D multibody dynamics considering chosen imperfections

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Software for simulating multibody systems' (MBS) dynamics allows engineers to study and investigate mechanical and mechatronic systems' motion. It enables them to generate and solve virtual 3D models to predict and visualise motion, coupling forces, or stresses [2].

Because simulation of MBS dynamics is usually quite time demanding, MBS software must be efficient in calculations if it is to be employable in practice. This work focuses on joint formulations and their evaluation efficiency, as these expressions are evaluated many times during simulations. Moreover, the contribution investigates effects of joint imperfections (friction and clearance) on the resulting motion of a 2-D slider-crank mechanism.

The mathematical basis of the in-house developed MBS software is formed by Lagrange's equations of the first kind that represent a set of findings achieved by applying Hamilton's principle. The dynamics of a spatial system of n interconnected rigid bodies can be described according to [3] by a system of $6n + r$ differential-algebraic equations (DAEs)

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_\mathbf{q}^\mathrm{T} \\ \mathbf{C}_\mathbf{q} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_e + \mathbf{Q}_v \\ \mathbf{Q}_d \end{bmatrix}, \tag{1}$$

where $\mathbf{M}$ is the system mass matrix, $\mathbf{q}$ is the vector of generalised coordinates of all bodies consisting of generalised coordinates $\mathbf{q}^i = [\mathbf{R}^i, \boldsymbol{\Phi}^i]^\mathrm{T}$ of particular bodies ($\mathbf{R}^i$ are the absolute Cartesian coordinates and $\boldsymbol{\Phi}^i$ are the orientation angles), $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers, $\mathbf{Q}_e$ is the vector of generalised applied forces, and $\mathbf{Q}_v$ is the quadratic velocity vector. The constraint Jacobian matrix $\mathbf{C}_\mathbf{q}$ and the vector $\mathbf{Q}_d$ are products of time derivatives of kinematic constraints (kinematic relationships describing mechanical joints or specified motion trajectories) represented by an algebraic system of r constraint equations

$$\mathbf{C}(\mathbf{q}, t) = \mathbf{0}, \tag{2}$$

where $\mathbf{C}$ is the constraint vector. If system (2) is twice partially differentiated with respect to time t, the kinematic acceleration equations are obtained, forming the last r equations in (1), where the vector $\mathbf{Q}_d$ can be expanded to

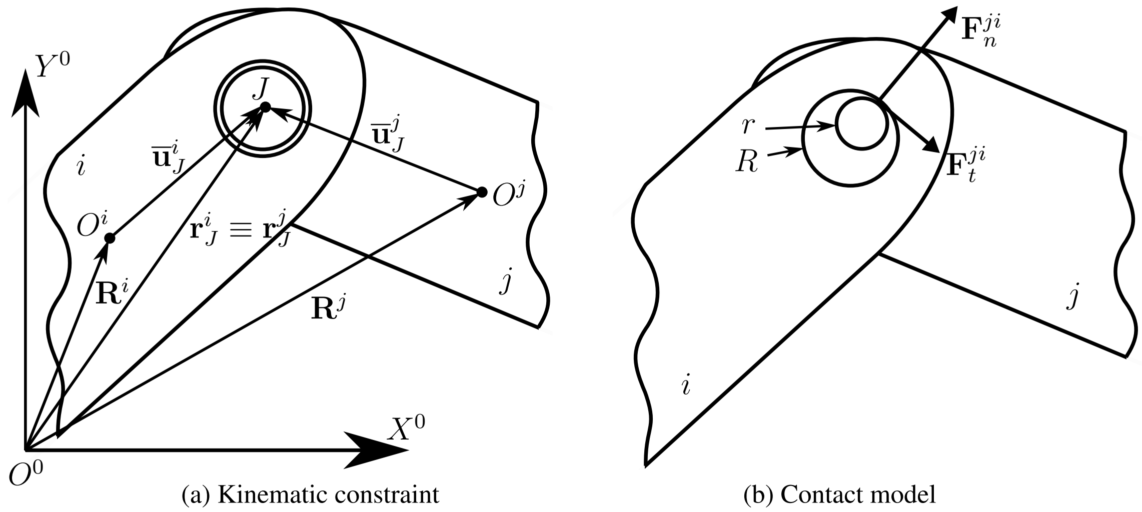
$$\mathbf{Q}_d = -\mathbf{C}_{tt} - (\mathbf{C}_\mathbf{q}\dot{\mathbf{q}})_\mathbf{q}\dot{\mathbf{q}} - 2\mathbf{C}_{\mathbf{q}t}\dot{\mathbf{q}}\,. \tag{3}$$

Lower indices $\cdot_t$ and $\cdot_\mathbf{q}$ indicate partial derivatives with respect to time and with respect to generalised coordinates.

One of many possible approaches to solve the problem of MBS dynamics is, for instance, to convert system (1) of DAEs of index 1 to the underlying system of ordinary differential equations (ODEs) and subsequent usage of some standard ODE solver. The transformation of (1) to a system of ODEs can be done via the elimination of Lagrange multipliers, obtaining

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}\mathbf{C}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{C}_{\mathbf{q}}\mathbf{M}^{-1}\mathbf{C}_{\mathbf{q}}^{\mathrm{T}})^{-1}[\mathbf{Q}_d - \mathbf{C}_{\mathbf{q}}\mathbf{M}^{-1}(\mathbf{Q}_e + \mathbf{Q}_v)] + \mathbf{M}^{-1}(\mathbf{Q}_e + \mathbf{Q}_v). \tag{4}$$

System (4) of ODEs can be solved directly, and it is possible to improve numerical accuracy via stabilization techniques, e.g., Baumgarte's stabilization, which can be found in [1].



(a) Kinematic constraint (b) Contact model

Fig. 1. Two approaches to modeling a revolute joint

There are two basic ways how to define a connection (usually called a joint) between two bodies. First approach is to use kinematic constraints representing restrictions on the relative motion between connected bodies. For instance, a revolute joint visualised in Fig. 1a eliminates relative translation between two bodies, or a prismatic joint eliminates relative rotation between two bodies. There are several other joint constraints and some of them have realistic meaning only in 3-D space such as a cylindrical joint or a spherical joint. All joints defined as kinematic constraints are included in the constraint vector $\mathbf{C}$. In Fig. 1a, a revolute joint constraint is visualised, where $\mathbf{R}^{i,j}$ are absolute coordinate vectors of linked bodies, $\overline{\mathbf{u}}_J^{i,j}$ are coordinate vectors of the joint J in local coordinate systems with origins $O^{i,j}$, and $\mathbf{r}^{i,j}$ are coordinate vectors of the joint in absolute coordinate system with origin O^0. The constraint equation is simple in case of the revolute joint: $\mathbf{r}_J^i - \mathbf{r}_J^j = 0$.

The idea of the second approach is in specifying contact forces that are generated when surfaces of two particular bodies touch each other. Similarly to the kinematic approach, there are different definitions of contact forces depending on the type of a joint. All joints that are represented by contact forces are simply included the vector $\mathbf{Q}_e$ of generalised applied forces. Fig. 1b contains visualization of a revolute joint with contact forces $\mathbf{F}_n^{ji}$ and $\mathbf{F}_t^{ji}$ applied on the body i (contact forces applied on the body j are not shown, as they can be trivially derived by applying Newton's third law), where R is the radius of the bearing, and r is the radius of the journal.

The advantage of the contact approach lies in the fact that it naturally contains parameters of joint imperfections (friction and clearance). In case of kinematic constraints, additional equation defining friction force has to be included in the model, and it is obviously impossible to include joint clearance. In this work, the first approach is used only in case of ideal joints without any imperfections. If it is needed to incorporate joint imperfection, the second approach is utilised.

During the implementation of any joint kinematic constraint to MBS software, it is necessary to have access not only to the constraint vector $\mathbf{C}$ but mainly to its derivatives $\mathbf{C_q}$ and $\mathbf{C_{qq}}$, while the second expression is included in the modified form of (3). Equation (3) also contains derivatives with respect to time t, but since joint kinematic constraints are usually scleronomic, all time derivatives of $\mathbf{C}$ are zero. Nevertheless, evaluation of $\mathbf{C_q}$ and $\mathbf{C_{qq}}$ is generally quite time consuming operation and represents a significant portion of total simulation time. On the other hand, evaluation of contact forces is not as time consuming as kinematic constraints, however, those forces bring important non-linearities to the mathematical model. This has to be taken into account when choosing and setting the numerical solver, and simulating non-linear system is generally more time consuming than solving the linear one.

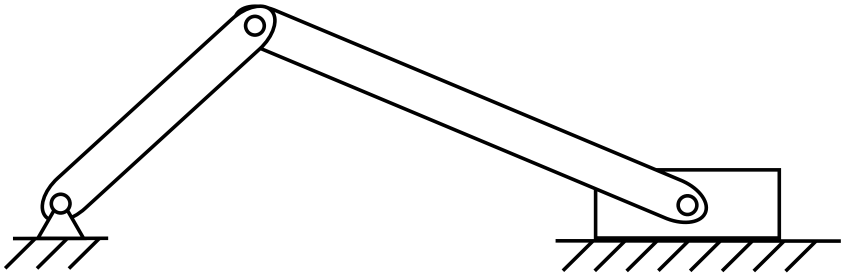


Fig. 2. A 2-D slider-crank mechanism

This work compares time demand for simulating the motion of a 2-D slider-crank mechanism, which is depicted in Fig. 2, without imperfections in two different model versions. Both versions differ in the approach used for modeling the joint between the crank and the connecting rod. All other joints (revolute between the frame and the crank, revolute between connecting the rod and the slider, prismatic between the slider and the frame) are defined as kinematic constraints in both models.

Furthermore, the effects of imperfections of the joint linking the crank and the connecting rod are investigated in a separate study. Specifically, the influence of the clearance size $R-r$ and the amount of friction are examined during various working conditions defined by the angular velocity of the crank.

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