Fast Intensity Distribution Functions for Soft and Hard Edged Spotlights

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ABSTRACT

The purpose of this paper is twofold: to propose two fast distribution functions for spotlights and to use terminology used in stage lighting to model these luminaries. In OpenGL and other API's the original Warn model is used where the light distribution is computed using a power function. In professional modeling tools, a linear or a cubic function is often used. We propose the use of two different quadratic functions instead that will make the computation involved faster than using the power function or a cubic function. Moreover it will be more flexible than using a linear function. These functions can be used to model both hard and soft edged spotlights.

Keywords

Spotlights, Beam Angle, Field Angle, Luminous intensity distribution.

1. INTRODUCTION

Computer graphics is in many ways a question about handling and computing light in a proper way. When a lighting artist is lighting a scene for a computer animated movie, he usually makes use of several light sources, like key, fill and back light, in order to produce lighting that looks right [Ran00]. Often some kind of spotlight is used in this process. Warn [War83] showed how the light could be concentrated in the primary direction of the spotlight by using a power function. If the primary direction is denoted **s** and the direction to the pixel currently shaded is denoted **l** as shown in figure 1, then the diffuse intensity is modeled by a directional multiplier

$$D(\phi) = \cos(\phi)^C = (\mathbf{s} \bullet \mathbf{l})^C \qquad (1)$$

where the exponent C provides control over the concentration of the light. The concentration can be delimited by a cone, where the intensity is set to zero outside the cone. This factor is then used when computing the diffuse and specular light using the Phong illumination model [Pho75]. As an example, the diffuse intensity of a pixel is computed as

$$I_d = D(\phi)(\mathbf{n} \bullet \mathbf{l}) \qquad (2)$$

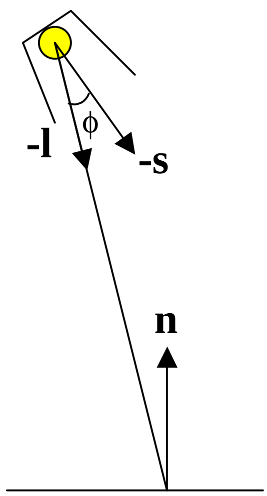

Figure 1. A spotlight has a main direction and a direction to the pixel being shaded.

The specular intensity is computed in a similar way, by multiplying the factor with the specular intensity.

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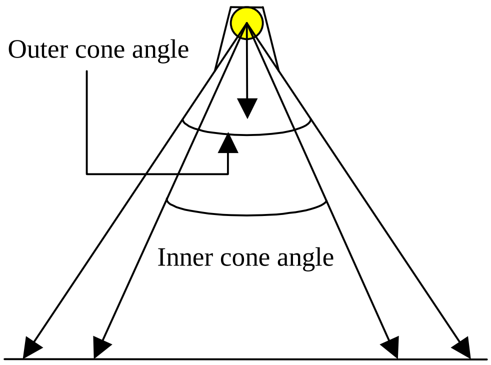

Figure 2. The intensity distribution is constant within the inner cone and drops off to zero, using a ramp function, when it reaches the outer cone.

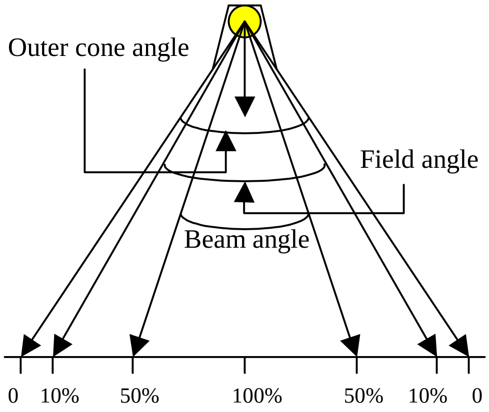

Figure 3. In stage lighting Beam and Field angles are used to describe the spotlight.

Even though the power function is relatively slow, this model is still used for several API's [Woo97, Vac96].

Another model is often used by professional modeling tools [Abo99] and in graphics hardware [Fer03]. The basic idea is that inside an inner cone, or hotspot, the concentration of light is constant. While outside this cone the intensity drops off smoothly down to zero where it reaches the outer cone. This ramp function is either a linear or cubic function. However, the cubic function produces more visually appealing results than the linear drop off. In this paper we will show that a quadratic function produces good results and we will also show how both soft and hard edge types of spotlights can be produced. Furthermore, we will show how terminology regarding characteristics of the beam distribution, used in stage lighting can be used for the proposed models. By using the proposed ramp functions it is possible to obtain a light distribution that mimics both soft and hard edge types.

Previous Research

More sophisticated light models have been developed by others. Barzel [Bar97] shows how lighting using the Warn model can be modified and controlled so that the shape of the light could be changed. Verbeck and Greenberg [Ver84] showed how the light distribution of a real light source could be measured and used for a virtual light source. The measured distribution is showed in a goniometric diagram. Goesele et al. [Goe03] showed how the luminous intensity distribution of flash lights and similar could be measured and accurately rendered. The problem with accurate measurements is also discussed by Albin and Peroche [Alb03]. Qing and Jizhou [Qin00] propose how interactive editing of light source intensity distribution and rendering using this luminaire could be done. Even though a physically measured light distribution is far better than a simple approximation, it will certainly take more time to compute. Nishita [Nis85] propose interpolation in a table where the luminous intensity data are given, for example, every ten degrees. However, for many real-time renderers this will still not be fast enough, especially if the rendering is done in software.

Stage Lighting

The Warn model is quite simple and the type of spotlights that can be simulated is rather limited. By using different functions it is possible to obtain light distributions that mimics both soft and hard edge types of spotlights. In stage lighting different types of luminaries are used in lighting for theaters and such [Fra99]. Flood-lights usually has no lens and spreads light in a broad angle. Other luminaries have a lens, which makes it possible to focus the light when gobos are used to make patterns in the beam. Some spotlights have very hard edges and can be useful for specials. This is when a single luminaire is brought up on a solo actor for a monologue or similar. Such a spotlight is sometimes called a follow spot. Profile spotlights can be used for these and they can produce both hard and soft edges. The Fresnel spotlight on the other hand produces only soft edges. These two are probably the most common types of spotlights. There are also a number of other spotlights that have characteristics similar to the Fresnel spotlight, but differ in some properties. An example of this is the Pebble Convex, which has characteristics similar to the Fresnel spotlight, but is considered better for specials. Another type of spotlight is the PARCAN, which has no spot size variation, but a powerful light output and is also considered good for specials.

Terminology

The characteristics of spotlights, i.e the luminous intensity distribution function, is often described

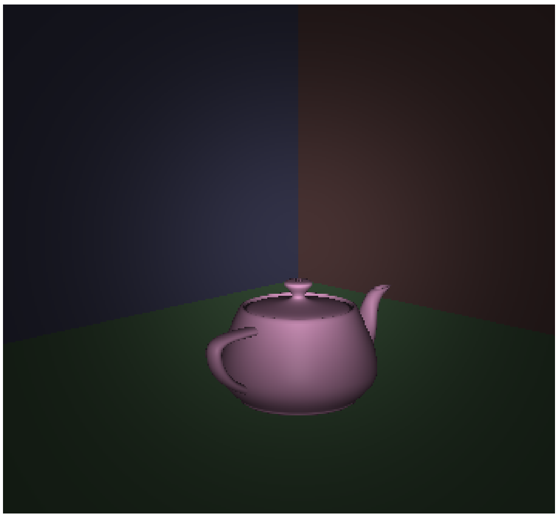

Figure 4. A scene rendered using Warns model with a power distribution function.

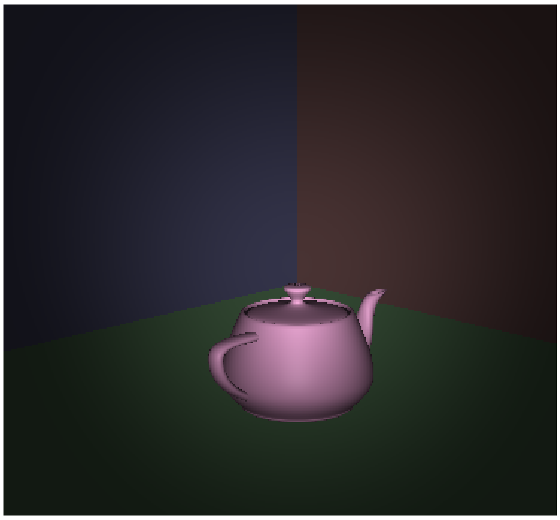

Figure 5. A scene rendered using the soft edge model.

using angles measuring the spread of light within the cone. Two important measures are: beam angle and field angle [Dor91]. The beam angle is the angle where the intensity has dropped off to 50% of the intensity along the beam axis, i.e. the main direction. The field angle is the angle where the intensity has dropped off to 10% of the intensity along the axis. These angles are shown in figure 3.

2. A SOFT EDGE SPOTLIGHT MODEL

The original Warn model produces a very soft edge spot similar to a Fresnel spotlight. The reason for this is that a power function drops off to close to zero rather quickly. A quadratic function can approximate this behavior quite well. A soft edge spotlight can be modeled by the quadratic function

$$D(\rho) = \frac{(\rho - \tau)^2}{(1-\tau)^2} \qquad (3)$$

where ρ is the cosine of the angle between the main direction, or the beam axis, and the pixel in question. Note that the corresponding angle is only half the angle we describe with the beam and field angles. In order to make the formulas easier to read, we have chosen to use variables for the cosines of the half of the angles in questions. Furthermore, we obtain the cosine of half the angles by the dot product and it is therefore easier to use this notation. Therefore, τ is the cosine of half the outer cone angle. The equation will yield zero when ρ is equal to τ and it will be one when ρ is equal to one. We can calculate a value for τ depending on the properties we would like our spotlight to have. This function flattens out when ρ comes closer to τ and therefore it will model a soft edge type of spotlight.

This simple model only allows us to set one of the mentioned angles. Either we set a predefined field angle or beam angle. The other can be computed as shown later. Figure 5 shows the result and it is clear that the edges are very soft as they are using the original Warn model, which is shown in figure 4.

A predefined field angle

If we choose to set the field angle with σ as the cosine of half the field angle then we have to compute τ for the distribution function. We set

$$\frac{1}{10} = \frac{(\sigma - \tau)^2}{(1-\tau)^2} \qquad (4)$$

And then solve for τ

$$\tau = \frac{10\sigma - \sqrt{10}}{10 - \sqrt{10}} \qquad (8)$$

The cosine of half the beam angle will similarly be computed by

$$\delta = \tau + (1-\tau) / \sqrt{2} \qquad (9)$$

A predefined beam angle

If we, on the other hand, choose to set the beam angle to δ, then it can be shown that

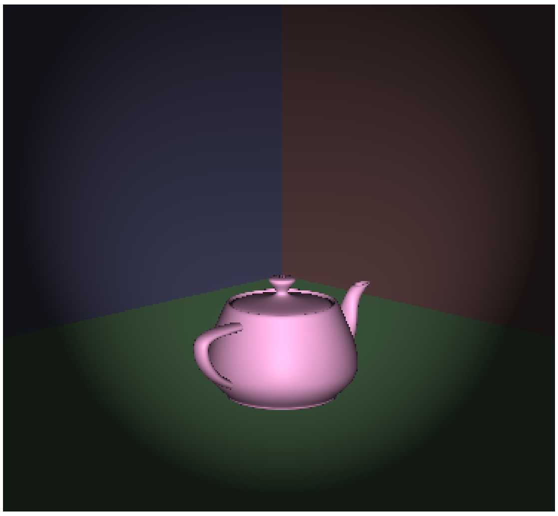

Figure 6. A scene rendered using the hard edge distribution function.

$$\tau = \frac{2\delta - \sqrt{2}}{2 - \sqrt{2}} \qquad (10)$$

In this case the cosine of half the field angle is

$$\sigma = \tau + (1 - \tau) / \sqrt{10} \qquad (11)$$

3. A HARD EDGE SPOTLIGHT MODEL

Hard or crisp edges can be obtained by a function that does not fade out and yield a function value close to zero when ρ becomes smaller. Instead the function has a rather high slope for ρ equal to τ. This can be obtained by the quadratic function

$$D(\rho) = 1 - \frac{(\rho - 1)^2}{(\tau - 1)^2} \qquad (12)$$

This distribution will be equal to one when ρ is equal to one and it will be equal to zero when ρ is equal to τ.

We derive the necessary computation for the case when we set the field angle

$$\frac{1}{10} = 1 - \frac{(\sigma - 1)^2}{(\tau - 1)^2} \qquad (13)$$

Hence

$$\sigma = 1 + \sqrt{\frac{9}{10}}(\tau - 1) \qquad (14)$$

And

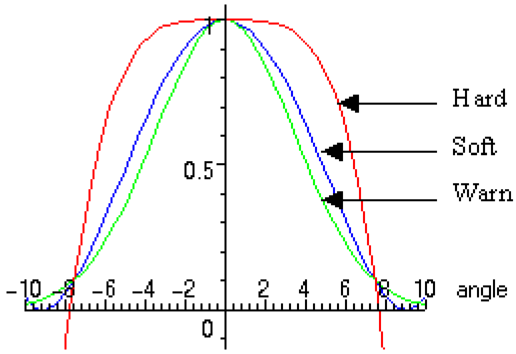

Figure 7. A Cartesian diagram showing the Warn distribution and both the proposed distributions used in the renderings.

$$\tau = \frac{10\sigma - 10 + 3\sqrt{10}}{3\sqrt{10}} \qquad (15)$$

The cosine of half the beam angle will then be

$$\delta = 1 + \frac{\sqrt{2}}{2}(\tau - 1) \qquad (16)$$

A predefined beam angle

If we wish to define the beam angle, then

$$\tau = \frac{2\delta - 2 + \sqrt{2}}{\sqrt{2}} \qquad (17)$$

And the cosine of half the field angle will then be

$$\sigma = 1 + \sqrt{\frac{9}{10}}(\tau - 1) \qquad (18)$$

Figure 6 shows a scene rendered using this approach. The edges are much harder than using the soft edge model.

4. RESULT

The proposed models are much faster than using an power function. The scenes in this paper were rendered by using a fragment shader and the Cg language. However it can be mentioned that 1 Billion calls to the power function in the C language (math.h) and the proposed method was conducted on a Pentium 4, 1.8 GHz and the power function took 300.27 seconds to execute, while the proposed method for soft edges took 6.29 seconds.

Figure 7 shows a Cartesian diagram over the Warn distribution function as well as the proposed functions used in the renderings. A field angle of 15° was used. Note that the Warn model and the proposed model for soft edges are quite similar. It can also be noted that the hard edge model does not flat out as much as the others do, which gives it its characteristic hard edge.

5. DISCUSSION

Linear functions are of course faster than the proposed quadratic functions. However, a linear ramp must be clamped when it reaches max intensity in the hot spot, i.e. the inner cone. This will yield Mach bands in the spot and should therefore be avoided. A cubic function can interpolate softer and is therefore better, but will be slower to use, due to a more complex computation.

The edge of the spot can be modified further with the proposed functions. This can be achieved by using an outer cone to delimit the distribution function. In practice, this can easily be done by setting the function to zero whenever ρ reaches a predefined threshold value that must be greater than τ.

It would have been preferable to be able to set both the field and beam angles. This is possible to some extent by reformulating the functions so that they depend on both σ and δ. However, it turns out that in the process of doing so, the maximum intensity is sometimes obtained for values of ρ greater than one, which is physically impossible. As an example, a typical Fresnel spotlight can have field angle of 15^o and a beam angle of 7^o. However, a field angle of 15^o will yield a beam angle of 9.8^o, so it will be quite close and sufficient for most needs, taking into account that we use an approximation of the luminous intensity distribution.

6. CONCLUSIONS

It has been shown that quadratic luminous distribution functions can be used to model both hard and soft edge spotlights. These will be faster to use than the original Warn model that use a power function and in practice only can be used to model soft edge spotlights. Moreover, it is faster than using a cubic function and gives better result than using a linear function that must be clamped. The proposed luminous intensity distribution functions can be used to design the spotlights so that they will have, either a given field angle or a given beam angle.

Further Research

It should be investigated how cubic polynomials could be used to make it possible to set both the field and beam angles while still being able to get both hard and soft edge spots. This is out of the scope of this paper, which deals with fast luminous distribution functions. But there is still a need for more accurate and correct distribution functions, even though they are more complex and expensive to compute.

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